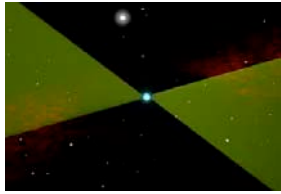


Astronomy 102
September 29, 2005

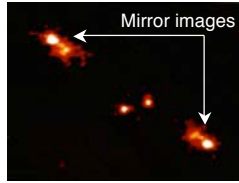


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The general theory of relativity.

- Einstein's general theory of relativity incorporates gravity.
- The motion of particles and light is governed by the curvature of space-time.
- The curvature of space-time is determined by the location and masses of the objects that are present.
- Einstein's general theory of relativity has been tested extensively.



Gravitational lens in galaxy cluster AC114.
Richard Ellis and Ian Smail
Hubble Space Telescope.

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Einstein's field equation.

- "Our" theory of general relativity:

"Space-time, with its curvature, tells masses how to move; masses tell space-time how to curve."

- The mathematical form of the theory of general relativity

$$\frac{1}{2} \left[\frac{\partial^2 g_{\lambda\lambda}}{\partial x^\alpha \partial x^\alpha} - \frac{\partial^2 g_{\mu\lambda}}{\partial x^\alpha \partial x^\lambda} - \frac{\partial^2 g_{\lambda\mu}}{\partial x^\alpha \partial x^\lambda} + \frac{\partial^2 g_{\mu\mu}}{\partial x^\alpha \partial x^\alpha} \right] + g^{\eta\sigma} \left[\frac{\partial x^\eta}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} - \frac{\partial x^\sigma}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} - \frac{\partial x^\eta}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} + \frac{\partial x^\sigma}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} \right]$$

$$- \frac{1}{2} g^{\mu\nu} \left[\frac{\partial^2 g_{\lambda\lambda}}{\partial x^\mu \partial x^\mu} - \frac{\partial^2 g_{\lambda\lambda}}{\partial x^\mu \partial x^\lambda} - \frac{\partial^2 g_{\lambda\lambda}}{\partial x^\lambda \partial x^\mu} \right] + g^{\eta\sigma} \left[\frac{\partial x^\eta}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} - \frac{\partial x^\sigma}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} - \frac{\partial x^\eta}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} + \frac{\partial x^\sigma}{\partial \xi^\alpha} \frac{\partial^2 \xi^\alpha}{\partial x^\lambda \partial x^\lambda} \right]$$

$$= -8\pi G \sum_n p_n \frac{dx_n^\alpha}{dt} \delta^3(x - x_n)$$

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The general theory of relativity.

- The general theory of relativity can be used to describe those regions in space where the gravitational force is very strong.
- As we will see today, a correct solution of Einstein's field equation leads to the prediction of the existence of black holes (although Einstein disagreed with this prediction).
- "Dark stars" were already predicted on the basis of Newton's theories, but required that photons experience the gravitational force. However, "dark stars" were later ruled out when photons did not appear to have mass (and thus were not effected by Newton's gravitational force).

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Escape velocities and massive stars.

- In 1783 John Mitchell started to study the escape velocity of particles from planets.
- If the launch velocity is too low, the particle will slow to a halt and fall back to the planet.
- If the launch velocity is higher than the escape velocity, the particle will slow down, but will escape the gravitational attraction of the planet.
- For the earth, the escape velocity is about 11 km/s (= 24,750 mi/h).



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Escape velocities and massive stars.

- Mitchell showed that the escape velocity has the following properties:
 - The escape velocity is proportional to the square root of the planet's mass.
 - The escape velocity is inversely proportional to the square root of the planet's radius.
- For a given mass, there will be a radius (or circumference) for which the escape velocity will be equal to the speed of light.
- Mitchell calculated that this critical circumference is 18.5 km for a star with a mass of our Sun.

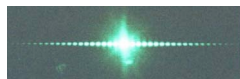
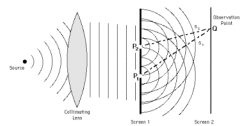


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Escape velocities and massive stars.

- John Mitchell thus predicted the possible existence of very compact stars, from which light could not escape.
- However, when the detailed nature of light became clear (its wave aspects and the fact that photons are massless), it did not appear reasonable to assume that light was influenced by gravity in the same manner as normal particles are.



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Karl Schwarzschild (1873 - 1916).

- Schwarzschild was interested in the physics of stars, and he solved Einstein's field equation for the region outside a massive spherical object.
- In the process he discovered some techniques, and ways of visualizing curved space-time, that benefited others who were doing research in general relativity.
- His solution of the Einstein field equation revealed the existence of black holes.



Karl Schwarzschild

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Warping of space-time and visualizing it.

- The warping of space-time can be visualized by using diagrams with additional dimensions. This creates the so-called embedding diagrams.
- An example of how one can illustrate the warping of two-dimensional space is the "deformed" rubber sheet.
- To a two-dimensional human being, it looks like his/her two-dimensional world is embedded in a three-dimensional world.

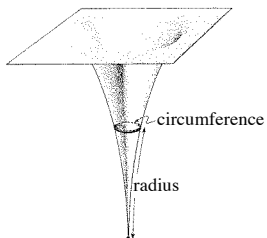


Figure from Thorne, *Black holes and time warps*

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Warping of space-time and visualizing it.

- How do we know that our space is warped?
- The flatness of space can be determined by looking at simple geometrical objects:
 - In flat space, the circumference of a circle is 2π times the radius.
 - In warped space, the circumference of a circle is less than 2π times the radius.
 - In flat space, two parallel lines will never intersect.
 - In warped space, two parallel lines may intersect.

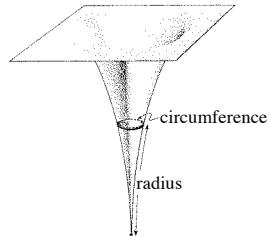


Figure from Thorne, *Black holes and time warps*

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Warping of space-time and checking the flatness of space.

2D human beings in their 2D world

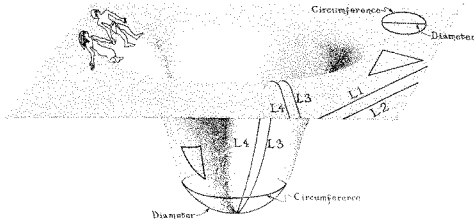


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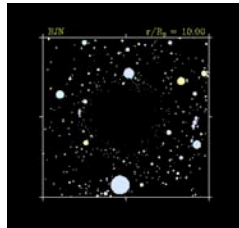
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Mid-lecture break. Let's circle a black hole!

- Homework set # 2 is due on Friday, September 30, at 8.30 am.
- Homework set # 3 will be available on Friday September 30, and will be due on Friday October 7 at 8.30 am.
- The first midterm exam is scheduled for Tuesday October 11. There will be no homework set due that week.

In orbit around a black hole!



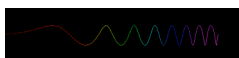
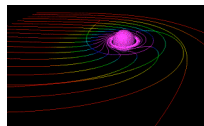
All movie frames, text, and computer codes are written, edited, and copyrighted by Robert J. Nemiroff

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The gravitational red shift.

- In the presence of time dilation, the frequency of light will change.
- Light coming from regions with strong gravity will be red shifted.
- The gravitational red shift of the light can be used to probe the gravitational strength.

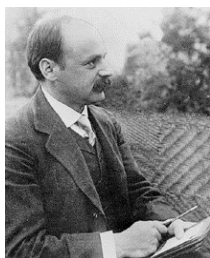


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Schwarzschild's solution to the Einstein field equations.

- Karl Schwarzschild was able to solve Einstein's field equations.
- He was able to calculate in detail the curvature of space-time outside a massive star.
- For a given, fixed star mass M , he examined how space and time are curved if the star is made smaller and smaller in size.
- If the star is made smaller than a certain critical size, the gravitational red shift of light predicted by his solution was infinite!



Karl Schwarzschild

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Visualizing the deformation of space around stars of fixed mass and different sizes.

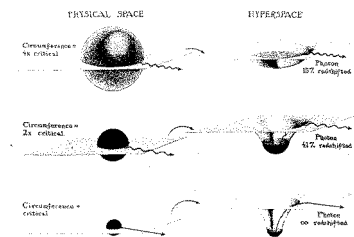


Figure from Thorne, *Black holes and time warps*

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Schwarzschild's solution of Einstein's field equation.

- If a star is made too small in circumference for a given mass, nothing can escape from it, not even light. This would be a black hole, and the critical size is the size of the black hole's horizon.
- The conclusion of Schwarzschild is similar to the 18th century idea of "dark stars". The theory of "dark stars" suggested that if light were subject to gravitational force, there could be stars from which light could not escape.
- The critical size of Schwarzschild's singularity turns out to be the same as that for the 18th century dark star.

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Schwarzschild's critical size.

- According to Schwarzschild's solution to Einstein's field equation for spherical objects, the critical circumference C_S is given by

$$C_S = \frac{4\pi GM}{c^2}$$

where $G = 6.67 \times 10^{-8} \text{ cm}^3/(\text{gm sec}^2)$ is Newton's gravitational constant, and $c = 2.9979 \times 10^{10} \text{ cm/sec}$ is, as usual, the speed of light (and $\pi = 3.14159\dots$).

$$C_S = 9.32 \times 10^{-28} M_{sun} \left(\frac{M}{M_{sun}} \right) = 1.865 \times 10^6 \left(\frac{M}{M_{sun}} \right)$$

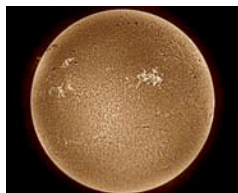
Note: units of C_S in cm.

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Schwarzschild's solution of Einstein's field equation.

- Schwarzschild's solution can be used to predict the red shift of solar light.
- The predicted effect is two parts in a million.
- The predicted effect could not be verified due to the state of technology in the early 20th century.
- It was not until around 1960 that the technology was accurate enough to measure this effect.
- The prediction was verified by Jim Brault at Princeton.



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The prediction of black holes.

- The solution obtained by Schwarzschild predicted the existence of black holes.
- Einstein did not accept this prediction. He and Arthur Eddington, a British astrophysicist, did not like the smell of black holes.
- The controversy did not get a lot of attention since Einstein and Eddington were considered the experts on general relativity.



Albert Einstein



Sir Arthur Eddington (1882-1944)

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The prediction of black holes.

- Einstein viewed a star of any size as a cluster of particles; each particle orbiting the center of the star.
- When the size of the star decreases, the orbital velocity of these particles will increase.
- Einstein calculated that when the radius of the star would be 1.5 times the Schwarzschild radius, the orbital speed would be the speed of light.
- Since only light can move with the speed of light, such a star could not exist, Einstein argued.



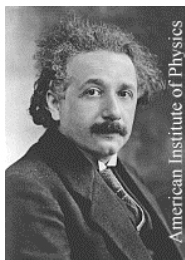
Albert Einstein in his study at home in Princeton, New Jersey, in 1938.

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The prediction of black holes.

- Einstein also showed that if the star's radius reaches a value of 1.125 times the Schwarzschild radius, the internal pressure of the star would become infinite.
- This conclusion was one more indication, according to Einstein, that black holes could not exist.
- Einstein's conclusions about the model of the star he was considering were correct. But maybe the model of the star is not correct.

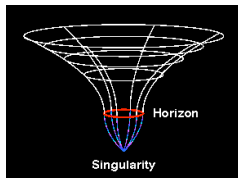


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The prediction of black holes.

- Although Einstein showed that no force can balance the gravitational force when a star approaches the critical circumference, this does not mean that such an object can not exist for a brief period.
- When the star reaches this point, a catastrophic implosion will occur and a black hole will be created.
- All matter will reside in one point: the Schwarzschild singularity.



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Quotes from Albert Einstein.

- "If the facts don't fit the theory, change the facts."
- "Only two things are infinite, the universe and human stupidity, and I'm not sure about the former."
- "God doesn't play dice."
- "If we knew what it was we were doing, it would not be called research, would it?"
- "Teaching should be such that what is offered is perceived as a valuable gift and not as a hard duty."

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We are done for today.
See you next Tuesday!



A classic planetary nebula, the Cat's Eye (NGC 6543), represents a final, brief yet glorious phase in the life of a sun-like star. Credit: NASA, ESA, HEIC, and the Hubble Heritage Team.

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