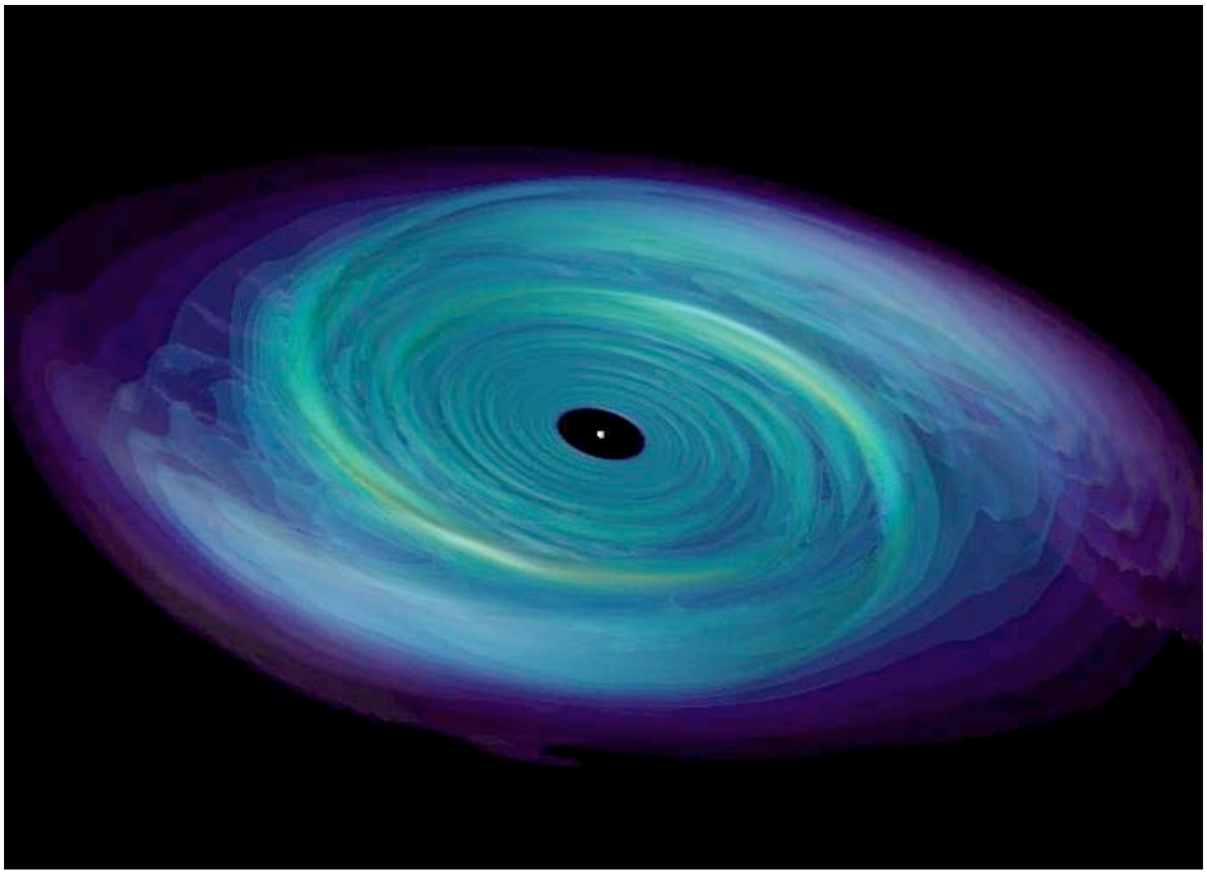
Astronomy 102
October 4, 2005



Accretion Disk Simulation. Credit: Michael Owen, John Blondin (North Carolina State Univ.)

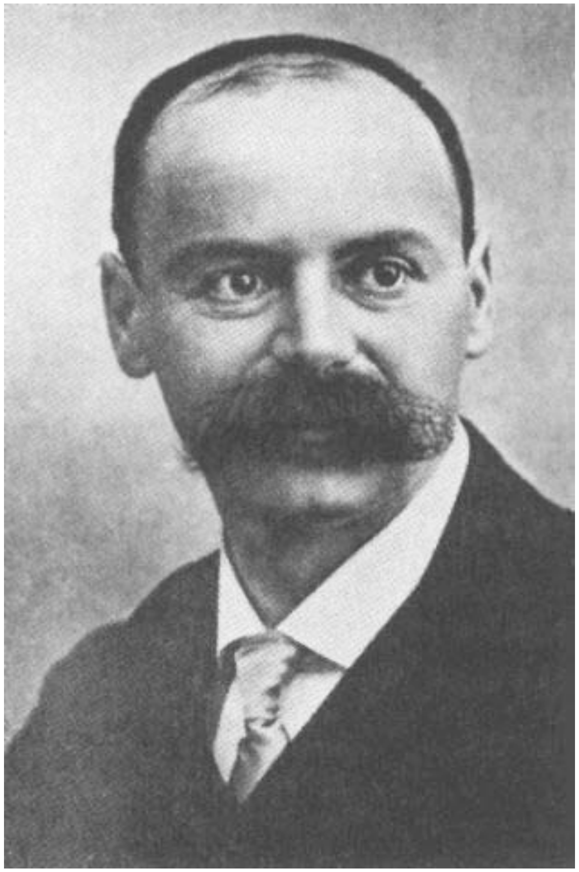
Karl Schwarzschild (1873 - 1916).

- In 1916 Schwarzschild read Einstein's paper on general relativity. Schwarzschild was interested in the physics of stars, and had a lot of spare time between battles on the Russian front, so he solved Einstein's field equation for the region outside a massive spherical object.
- Schwarzschild died on the front in 1916 at age 43.

Karl Schwarzschild

Karl Schwarzschild (1873 - 1916).

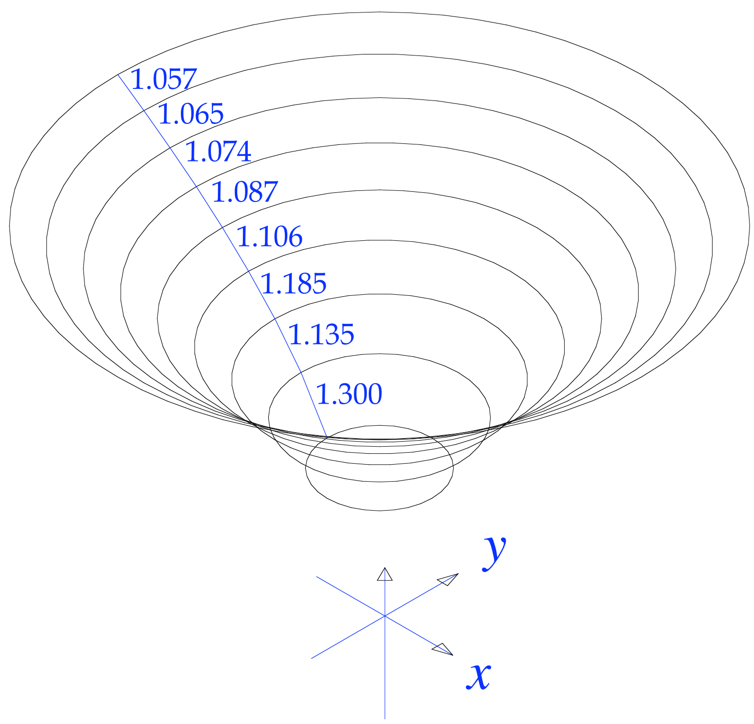
- The solution of Einstein's field equation had many interesting features:
  - The prediction of space warping in regions of strong gravity, and the invention of embedding diagrams to visualize it.
  - The verification of the gravitational time dilation, just as Einstein had pictured it.
  - The prediction of black holes, though this was not recognized at the time.



Karl Schwarzschild

Warped space around a black hole.

- To visualize warped space, hyper space was introduced.
- Consider the diagram on this page, were circles that differ in circumference by 2π are shown.
- In flat space, the different circles would have radii that differ by 1 length unit.
- In warped space, the difference in radii would increase the larger the warping. We can visualize this by offsetting these circles in the vertical direction with respect of to each other.
- The additional dimension that is used to offset the circles creates a space that is called hyper space.



Hyperspace

 Department of Physics and Astronomy, University of Rochester

Hyper space around a star.

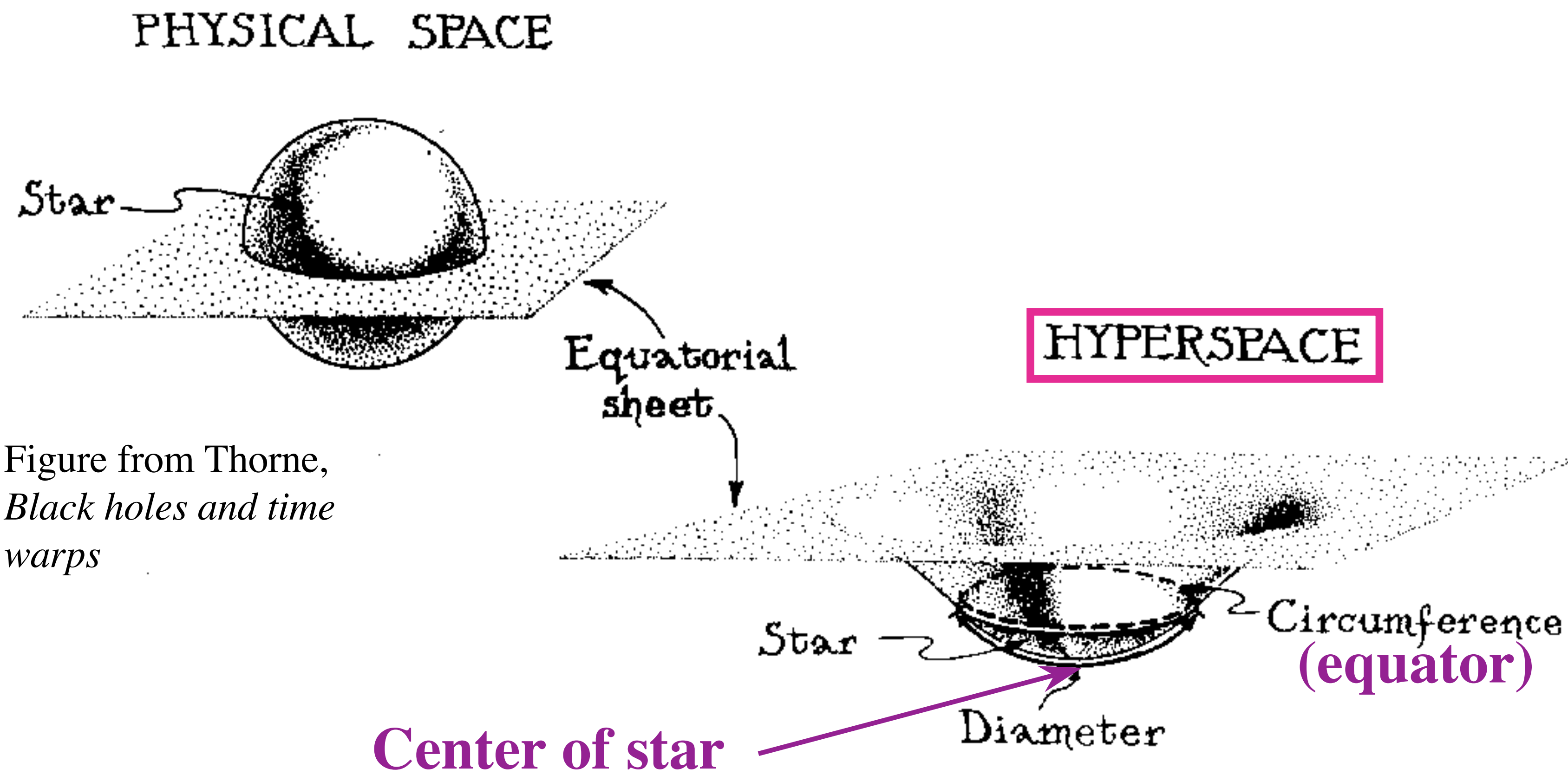


Figure from Thorne, *Black holes and time warps*

Hyper space around a star as function of star size.

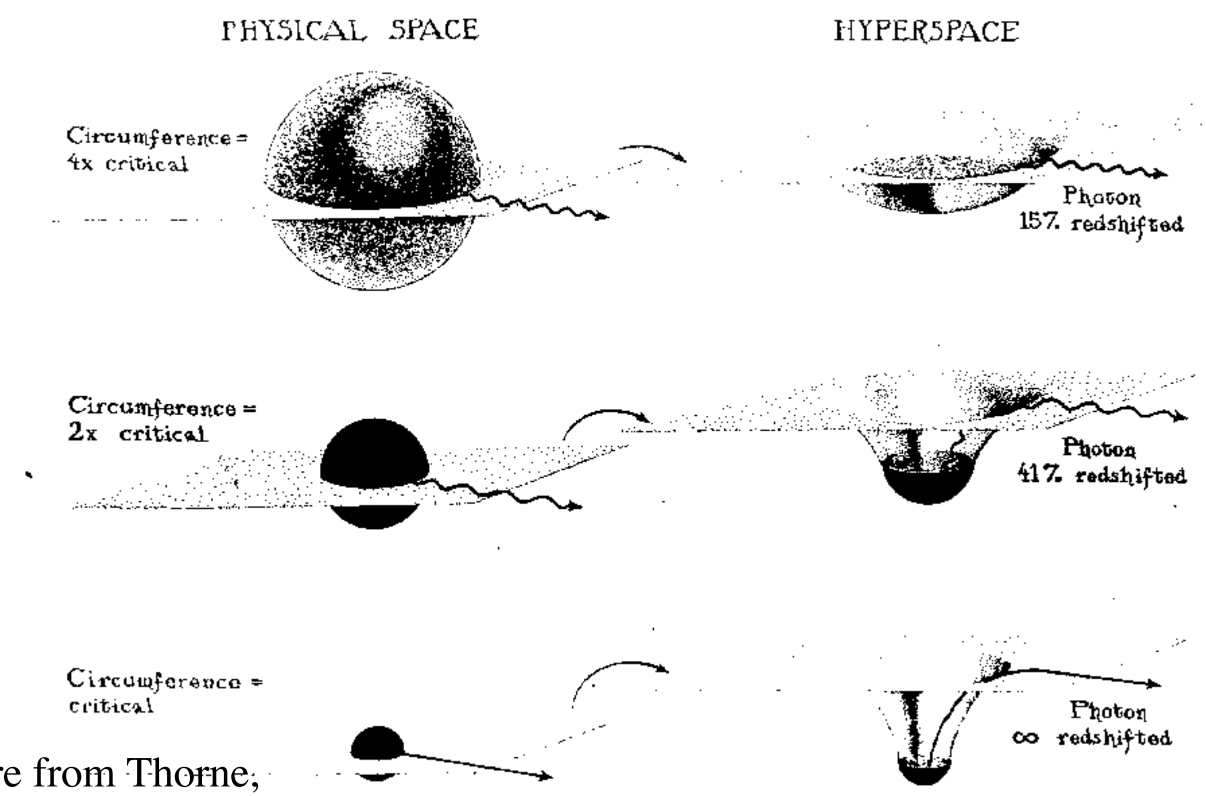


Figure from Thorne, *Black holes and time warps*

Schwarzschild's singularity.

- If the star is made smaller than a certain critical size, the gravitational red shift of light (time dilation, remember) predicted by his solution was infinite! In this case, a singularity is formed.
- The singularity is a black hole, and the critical size is the size of the black hole's horizon.
- The critical size of the Schwarzschild's singularity turns out to be the same as that for the 18th century dark star.

Schwarzschild's circumference.

- According to Schwarzschild's solution to Einstein's field equation for spherical objects, the gravitational red shift becomes infinite (i.e. time appears to a distant observer to stop) if an object having mass *M* is confined within a sphere of circumference C_S, given by

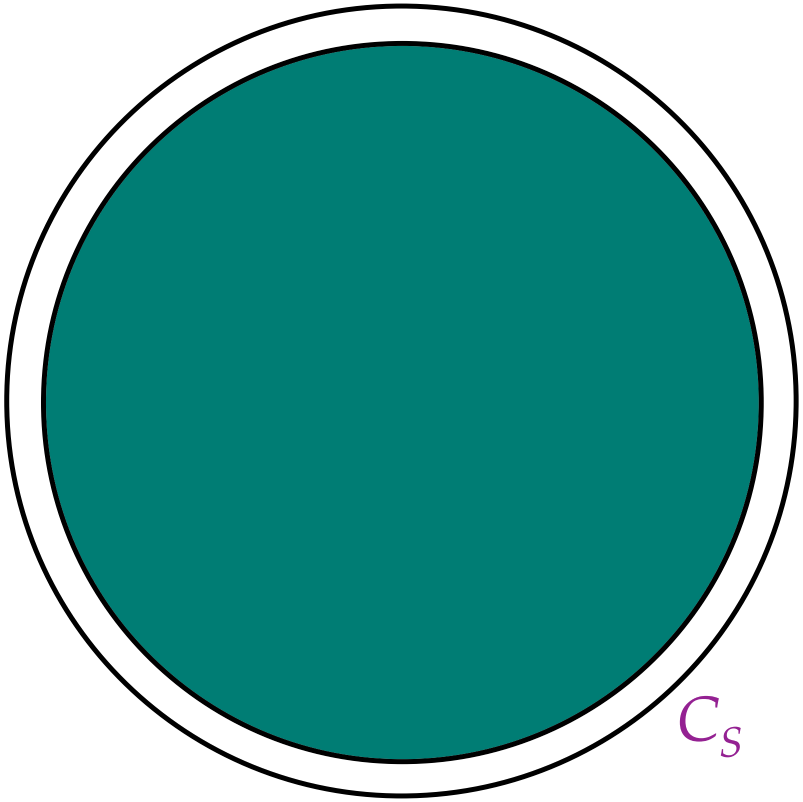
$$C_S = \frac{4\pi GM}{c^2}$$

where $G = 6.67 \times 10^{-8}$ cm^3/(gm sec^2) is Newton's gravitational constant, and $c = 2.9979 \times 10^{10}$ cm/sec is, as usual, the speed of light (and $\pi = 3.14159....$).

The event horizon.

- Any object with mass *M*, and circumference smaller than C_S, would not be able to send light (or anything else) to an outside observer -- that is, it would be a black hole.

- The sphere with this critical circumference is what we have been calling the **event horizon**, or simply the **horizon**, of the black hole.



Applying Schwarzschild's solution.

- Problem: What is the horizon circumference of a black hole with the same mass as the Earth ($M = 6.0 \times 10^{27}$ gm)?

- Solution:

$$C_S = \frac{4\pi GM}{c^2}$$

$$= \frac{4 \times 3.14 \times 6.67 \times 10^{-8} \ \frac{\mathrm{cm}^3}{\mathrm{s}^2\mathrm{gm}}}{\left(3.00 \times 10^{10} \ \frac{\mathrm{cm}}{\mathrm{s}}\right)^2} \times 6.0 \times 10^{27} \ \mathrm{gm}$$

$$= 5.6 \ \mathrm{cm}$$

Applying Schwarzschild's solution.

- Problem: What is the mass of a black hole that has a horizon circumference equal to that of the Earth ($4.0\mathrm{x}10^9$ cm)?

- Solution: First, rearrange the Schwarzschild's formula

$$C_S = \frac{4\pi GM}{S^2}$$

$$\frac{S^2}{4\pi G} C_S = \frac{4\pi GM}{S^2} \frac{S^2}{4\pi G}$$

$$\frac{C_S S^2}{4\pi G} = M$$

Applying Schwarzschild's solution.

- Problem: What is the mass of a black hole that has a horizon circumference equal to that of the Earth ($4.0\text{x}10^9$ cm)?

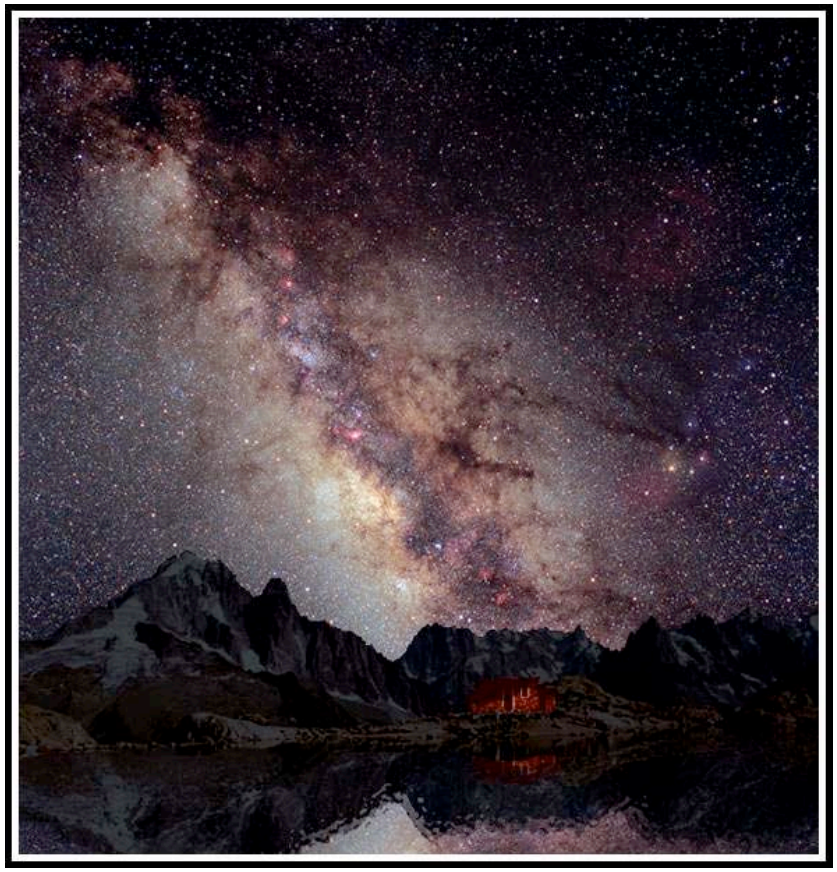
- Solution (continued): Now, plug in the numbers!

$$M = \frac{C_S c^2}{4\pi G} = \frac{4 \times 10^9 \text{cm} \times \left(3.00 \times 10^{10} \frac{\text{cm}}{\text{s}}\right)^2}{4 \times 3.14 \times 6.67 \times 10^{-8} \frac{\text{cm}^3}{\text{s}^2\text{gm}}} =$$

$$= 4.3 \times 10^{36} \text{gm} =$$

$$= 4.3 \times 10^{36} \text{gm} \times \frac{1\ M_{\text{sun}}}{2.0 \times 10^{33}\ \text{gm}} = 2.15 \times 10^3\ M_{\text{sun}}$$

Mid-lecture Break.
Enjoy the view of our Milky-Way.



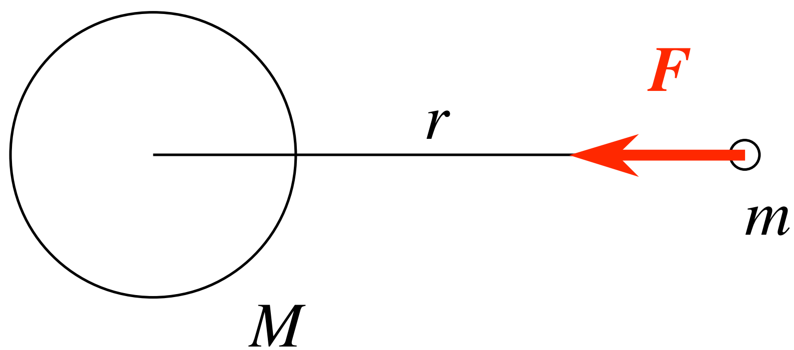
The Milky Way Over the French Alps
Credit & Copyright: Marc Sylvestre (Universia)

Singularities in physics, math, and astronomy.

- A formula is called **singular** if, under certain conditions, the result of a calculation is infinity or not well defined. The particular condition under which this happens is called the **singularity**.
- Singularities often arise in the formulas of physics and astronomy. They usually indicate either:
  - Invalid approximations -- not all of the necessary physical laws have been accounted for in the formula (no big deal), or
  - That the singularity is not realizable (also no big deal), or
  - That a mathematical error was made in obtaining the formula (just plain wrong).

Singularities in physics, math, and astronomy.

- An example of a classical physics law with a singularity is Newton's law of gravitation.



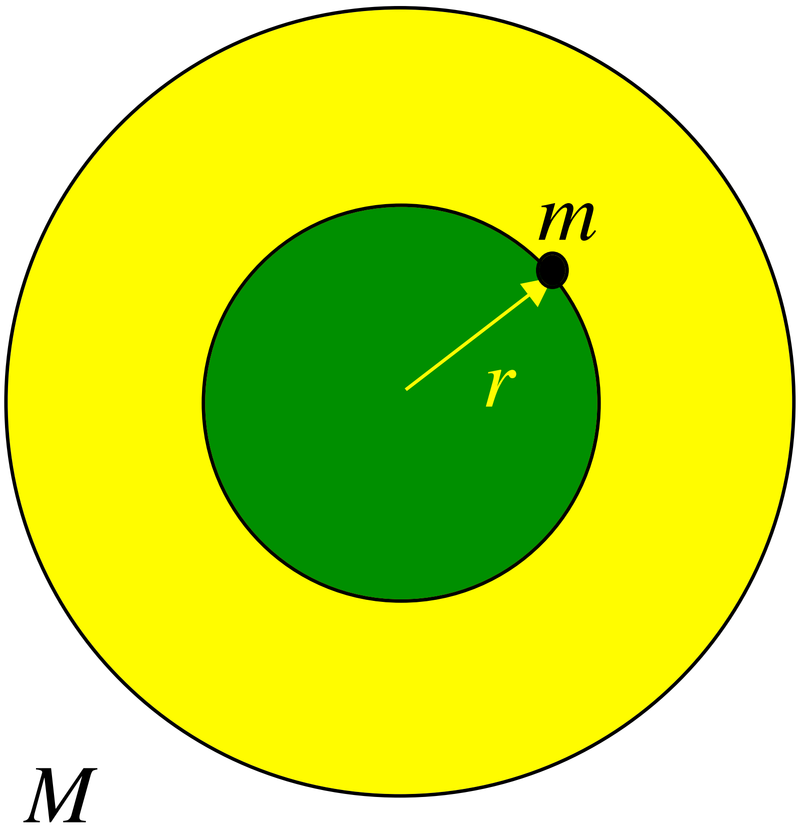
$$F = \frac{GMm}{r^2}$$

Here, r is the distance between the centers of the two spherical masses. Note: a spherical mass exerts force as if its mass is concentrated at its center.

- If r were zero, the force would be infinite!

Singularities in physics, math, and astronomy.

- This singularity is not real however:
  - The mass really isn't concentrated at a point.
  - A spherical shell of matter does not exert a net gravitational force on a mass inside it.
  - Consider mass m inside mass M. The outer (yellow) matter's forces on m cancel out, and only the inner (green) matter exerts a force. As m gets closer to the center ($r \rightarrow 0$), the force gets smaller, not larger.



The Schwarzschild's singularity.

- Schwarzschild's solution to the Einstein field equation was demonstrated to be correct - the Schwarzschild's singularity is not the result of a math error.
- Most physicists and astronomers assumed that the singularity would not be physically realizable (just like the singularity in Newton's law of gravitation) or that accounting for other physical effects would remove it.
- Einstein (1939) eventually tried to prove this in a general relativistic calculation of stable (non-collapsing or exploding) stars of size equal to the Schwarzschild circumference.
- He found that this would require **infinite gas pressure**, or **particle speed greater than the speed of light**, both of which are impossible.

The Schwarzschild's singularity.

- Einstein's results show that a stable object with a singularity cannot exist.
- From this, he concluded (incorrectly) that this meant the singularity could not exist in nature.
- Einstein's calculation was correct, but the correct inference from the result is that gas pressure cannot support the weight of stars similar in size to the Schwarzschild circumference.
- If nothing stronger than gas pressure holds them up, such stars will collapse to form black holes -- the singularity is real.
- The only pressure that can be stronger than gas pressure is degeneracy pressure.

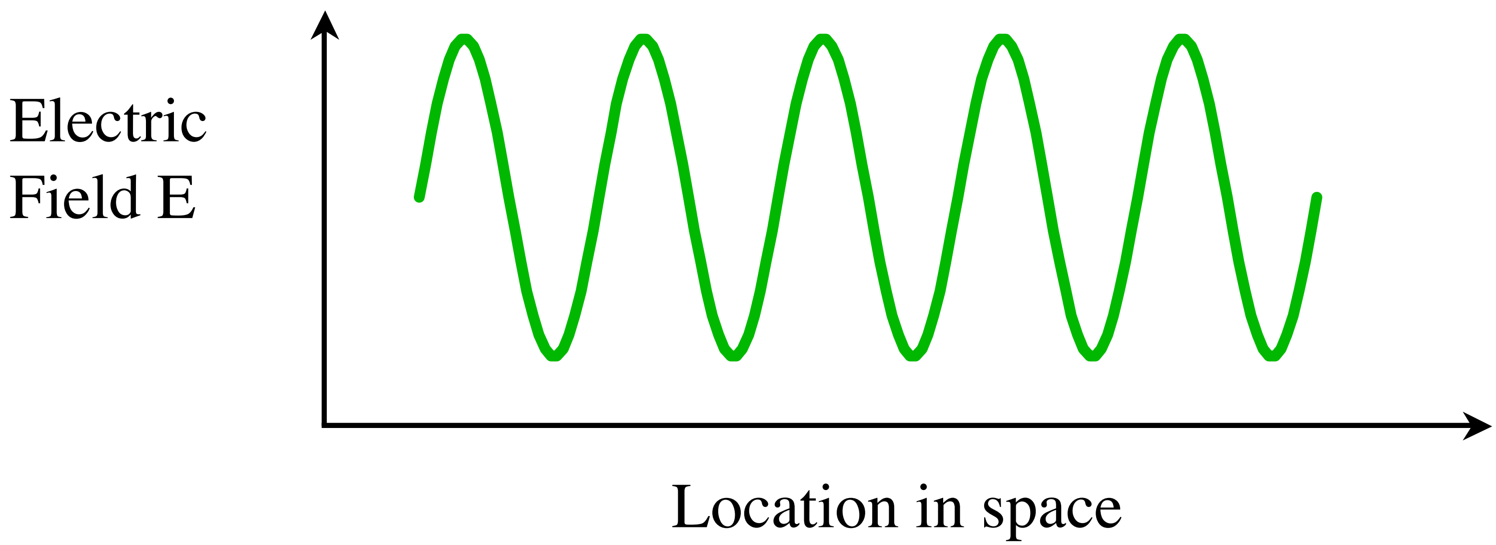
Degeneracy pressure.

- To understand degeneracy pressure we have to understand a concept from quantum mechanics called the wave-particle duality:
  - All elementary particles from which matter and energy are made (including light, electrons, protons, neutrons...) have simultaneously the properties of particles and of waves.
  - Which property they display depends upon the situation they're in.
- Degeneracy pressure consists of a powerful resistance to compression that's exhibited by the elementary constituents of matter when these particles are confined to spaces small enough to reveal their wave properties.

The wave-particle duality.

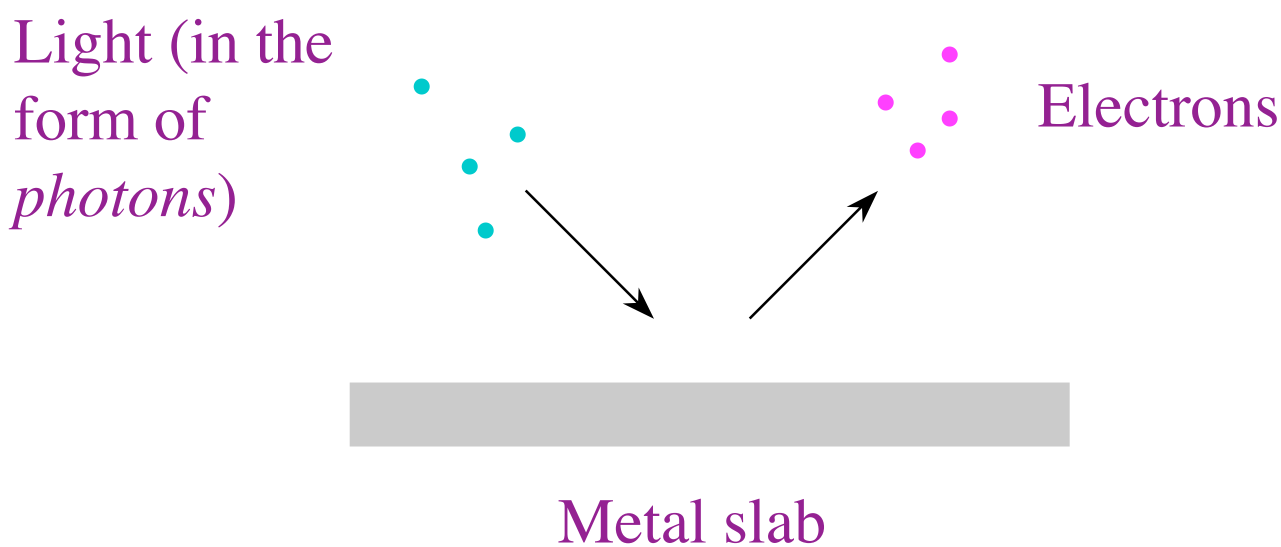
- Particles exist only at a point in space.

Waves extend over a region of space.



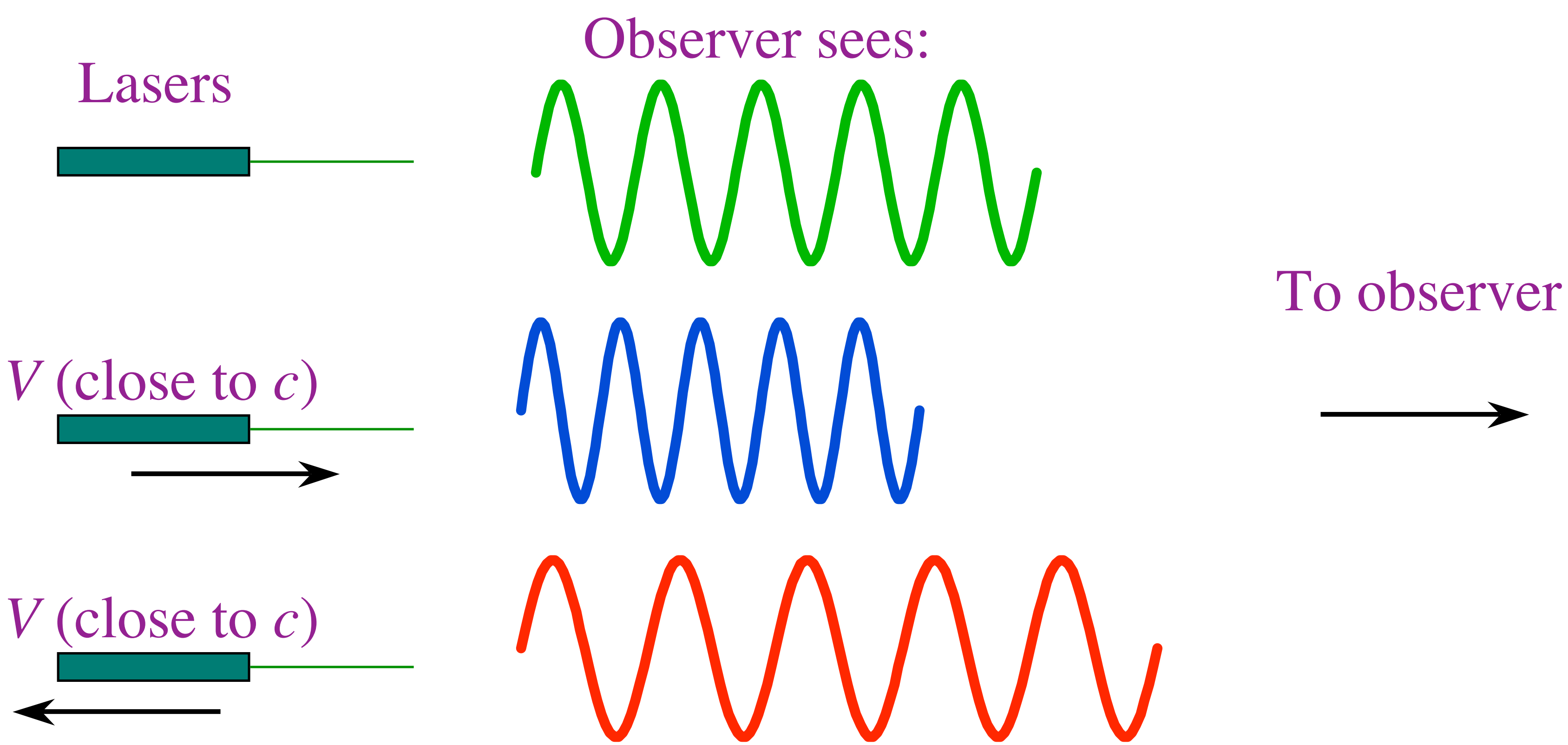
The wave-particle duality: light.

Under certain conditions, light exhibits particle properties. A good example is the photoelectric effect. The explanation of this effect in 1905 won Einstein the 1921 Nobel Prize in physics.



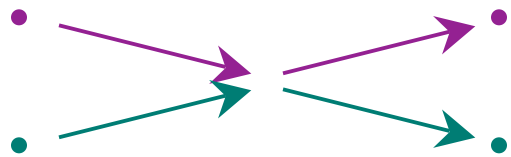
The wave-particle duality: light.

Under certain conditions, light exhibits wave properties. A good example is the Doppler effect.

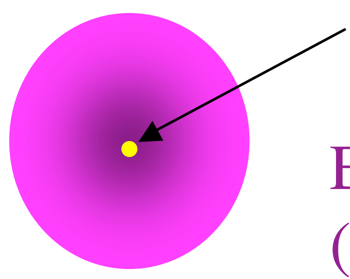


The wave-particle duality: electrons.

- Electrons behave like particles in elastic collisions between free electrons are "elastic" (they behave like billiard balls).



- Electrons confined to atoms behave like waves.



The wave-particle duality: when do particles behave like waves?

- All the elementary constituents of matter have both wave and particle properties.

- If a subatomic particle (like an electron, proton or neutron) is confined to a very small space, it acts like a wave rather than a particle.

- How small a space?
  - The size of an atom, in the case of electrons (about 10^{-8} cm in diameter).
  - A much smaller space for protons and neutrons (about 10^{-11} cm diameter).
  - Generally, the more massive a particle is, the smaller the confinement space required to make it exhibit wave properties.

What are we? Particles or waves? Think about it between now and Thursday!