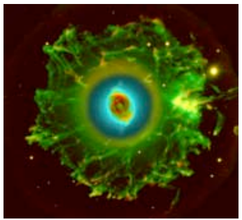
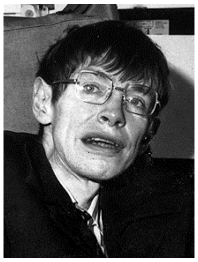
Astronomy 102.
November 15, 2003.

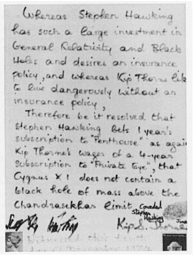
The Cat's Eye Nebula (NGC 6543) is one of the best known planetary nebulae in the sky. Its haunting symmetries are seen in the very central region of this stunning false-color picture, processed to reveal the enormous but extremely faint halo of gaseous material, over three light-years across, which surrounds the brighter, familiar planetary nebula. Made with data from the Nordic Optical Telescope in the Canary Islands, the composite picture shows emission from nitrogen atoms as red and oxygen atoms as green and blue shades.



Halo of the Cat's Eye. Credit: R. Corradi (Isaac Newton Group), D. Goncalves (Inst. Astrofisica de Canarias)

Stephen Hawking (1942): Made important contributions to the theories of Black Holes.





Whereas Stephen Hawking has such a large investment in General Relativity and Black Holes and desires an insurance policy, and whereas Kip Thorne lik
to live dangerously without an insurance policy,

Therefore be it resolved that Stephen Hawking bets 1 year's subscription to "Penthouse" as agai
Kip Thorne's wager of a 4-year subscription to "Private Eye", that Cygnus X1 does not contain a black hole of mass above the Chandrasekhar limit.

Conceded
Stephen Hawking

Kip S.

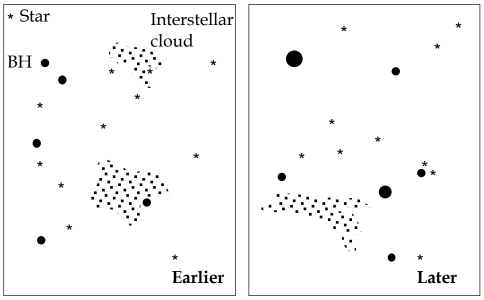
Stephen Hawking (1942): A brilliant physicist who overcame serious disabilities.

In 1962, a graduate student of Cambridge he was diagnosed as having amyotrophic lateral selerosis (ALS) and gave him few years to live. This gave him the motivation to take control and do something for his future. He began to research cosmology and black holes. Now, Stephen Hawking is still living and still continuing his research. His ALS slowed down and he is now in a wheelchair and speaks with the help of a computer.

The horizon area theorem.

- In 1970, Stephen Hawking used general relativity to prove a useful rule called the horizon area theorem:
  - The total horizon area in a closed system containing black holes never decreases. It can only increase or stay the same.
- Increases in total horizon area come from growth of black holes by collapse or accretion of “normal” matter, and by the coalescence of black holes.
- Illustration (next page): a closed-off part of the universe. As time goes on, the total area of all the horizons in this closed system increases, owing to the growth of black holes by collapse, accretion and coalescence.

Illustration of Hawking’s horizon-area theorem.



So why should that seem strange?

- The horizon-area theorem is simple (and intuitively obvious), but represents a puzzle at a deeper level when one reflects on the **heat** and **disorder** in the matter that forms or falls into a black hole.
  - Before: the matter is hot, and there are lots of particles sharing the heat among themselves in the form of their random motions. A complete description of the system would thus have different entries for position and velocity for each particle – a vast number of *numbers* required.
  - After: the system can be completely described by only three numbers, its mass, spin and charge. It’s orderly!
- The problem is, in all other natural processes **matter is never seen to go from a disorderly state to an orderly one all by itself.** This is in fact a law of thermodynamics…

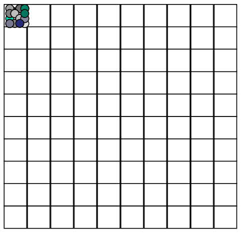
Entropy and the second law of thermodynamics.

- Compare these two statements:
  - The **horizon area theorem**:
    - The total horizon area in a closed system never decreases.
  - The **second law of thermodynamics**:
    - The total entropy of a closed system never decreases.
- **Entropy** = the logarithm of the number of ways all of the atoms and molecules in a system can be rearranged without changing the system’s overall appearance. A larger entropy means the system is more disorderly, or more “random.”
- Do black holes really have entropy as low as they seem to? Does the horizon area have anything to do with entropy?

Example of entropy: toys in a playroom (Thorne, pg. 424).

This playroom floor has 100 tiles, on which the kids can arrange 20 different toys. Parents prefer the toys to be kept in an extremely orderly configuration, with all the toys piled on one tile in one corner, as shown. There is only one such arrangement; the entropy of this configuration is thus the logarithm of 1, which is zero.

Extremely orderly: 20 toys on 1 tile.

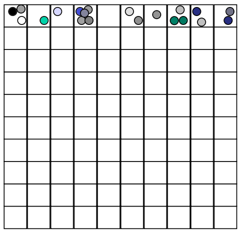


Number of equivalent rearrangements = 1; entropy = 0.

Example of entropy: toys in a playroom (Thorne, pg. 424).

Parents might even accept this somewhat less orderly configuration: 20 different toys on 10 specific tiles. But there are lots of different equivalent arrangements (e.g. swapping the positions of two toys on different tiles produces a different arrangement that’s still acceptable): 10^{20} of them, in fact, for an entropy value of $\log(10^{20}) = 20$.

Orderly: 20 toys on 10 tiles



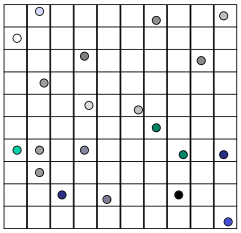
Number of equivalent rearrangements $=10^{20}$; “entropy” = 20.

Example of entropy: toys in a playroom (Thorne, pg. 424).

Of course, when the kids are done playing, the floor looks like this: 20 different toys spread randomly over 100 tiles. There are $100^{20} = 10^{40}$ different ways to do that; the entropy is 40.

And kids are like natural physical processes. Through their agency the room will not get less random; the entropy of the room full of toys and kids never decreases by itself.

Disorderly: 20 toys on 100 tiles



Number of equivalent rearrangements = 10^{40}; "entropy" = 40.

The second law of thermodynamics.

- Many consider the second law of thermodynamics to be the physical law least likely ever to be broken.
- This is because its consequences are so easily tested experimentally that you can see it verified all around you every day. Among the important consequences of the second law:
  - Heat never flows by itself from a lower temperature to a higher temperature.
  - Decreasing the temperature of one part of a closed system requires raising the temperature in other parts.
  - The mechanical work (*organized* energy) that can be done by a heat engine is always less than the available heat (*disorganized* energy).
- So why does black-hole formation look so much as if it is a violation of the Second Law?

Horizon area and entropy.

- Black holes form from large collections of atoms and molecules, with extremely large numbers of equivalent rearrangements (large entropy).
- What happens to the entropy of this matter when it falls into a black hole? This was a burning issue in 1972.
  - **Hawking** (and the rest of the relativists in 1972): The entropy vanishes. Black holes and their horizons are extremely simple objects, with only one possible configuration ("equivalent rearrangement") each: that means zero entropy. Black holes therefore violate the second law of thermodynamics. That doesn't matter very much, because you can't get the matter back out of the black hole anyway.

Horizon area and entropy.

- **Jacob Bekenstein** (a Princeton graduate student in 1972, and all alone on this side of the argument): The second law of thermodynamics hasn't been violated in any other physical situation; why give up so soon? The entropy of the ingredients may be preserved, in a form proportional to the horizon area. If hole has entropy it also must have a temperature, which I find is proportional to the strength of gravity at the horizon.
- **Hawking *et al.* (1972):** But that would mean that the horizon is a **black body** at non-zero temperature that obeys the laws of thermodynamics. Any such body must radiate light - as the hot filament in a light bulb does, for instance. But …. *nothing can escape from a black hole horizon*; how can it radiate?

Horizon area and entropy.

- **Hawking *et al.* (1972):** This contradiction implies that black holes cannot have entropy or temperature, and that they must violate the second law of thermodynamics.
- **Bekenstein (1972):** I can't think of any way for light, or anything else, to escape from a black hole; I admit that black holes can't radiate. But there must be something wrong with your viewpoint, because it must be possible for black holes to obey the laws of thermodynamics.

- And, actually, Zel'dovich already had thought of a way for horizons to radiate light, a year previously but unknown to these contestants until several years later.

Horizon area and entropy.

- **Hawking (1974):** Oops. There is a way for black holes to emit radiation: it involves quantum-mechanical processes near the black hole's horizon.
  - The emission of light is exactly as one would expect from a black body with temperature that increases as the strength of gravity at the horizon increases.
  - Therefore the black hole has entropy, which increases as the area of the horizon increases.
  - Therefore black holes obey the laws of thermodynamics. **Bekenstein is right after all.**

- The theory behind Hawking radiation includes the approach previously found by Zel'dovich as a special case.

Mid-lecture Break.
Remember …. Midterm Exam # 2.

- Exam # 2 will take place on Thursday November 17.
- Material covered on this exam is Lecture 11 - Lecture 18.
- There will a review of the material on Wednesday 11/16 between 7 pm and 9 pm in B&L 109.
- Make sure you bring a calculator and make sure it has fresh batteries.
- There will be no recitations on Thursday and next week.

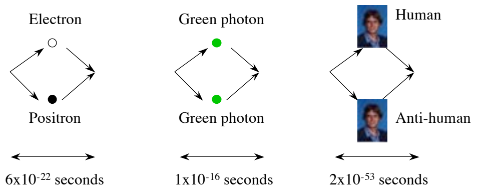
How black holes emit light and other particles.

- In subatomic matter, we see that energy conservation can be violated, though only temporarily and very briefly. This is expressed in one of Heisenberg's uncertainty principles:

$$\Delta E \times \Delta t = \hbar = 10^{-27} \text{ erg s}$$

- **Vacuum fluctuations**: the shorter the time interval Δt, the larger the energy ΔE that can be temporarily produced. For extremely short time intervals, enough energy can be borrowed from the vacuum (i.e. nothingness) to produce photons, or even massive particles.
- The particles thus made are called **virtual particles**. They vanish again at the end of the time interval Δt.
- Virtual particles are produced as **particle-antiparticle pairs**.

Examples of particle-antiparticle pairs made from the vacuum.



Electron

Positron

$6\text{x}10^{-22}$ seconds

Green photon

Green photon

$1\text{x}10^{-16}$ seconds

Human

Anti-human

$2\text{x}10^{-53}$ seconds

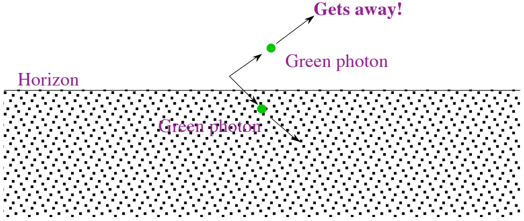
Energy = $1.6\text{x}10^{-6}$ erg

Energy = $8\text{x}10^{-12}$ erg

Energy = $4.5\text{x}10^{25}$ erg

How black holes emit light.

- Normally, virtual pairs vanish too quickly to be noticed, or to interact much with anything else.
- Near a black hole horizon: what if one of the pair falls in, and the other doesn't, and is aimed just right?

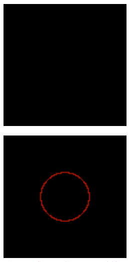


How black holes emit light.

- Details of black-hole emission, nowadays called **Hawking radiation**:
  - Virtual pairs, produced by vacuum fluctuations, can be split up by the strong gravity near a horizon. Both of the particles can fall in, but it is possible for one to fall in with the other escaping.
  - The escaping particle is seen by a distant observer as emission by the black hole horizon: **black holes emit light** (and other particles), though only in this weird way.
  - The energy conservation "debt" involved in the vacuum fluctuation is paid by the black hole itself: the black hole's mass decreases by the energy of the escaping particle, divided by c^2. The emission of light (or any other particle) costs the black hole mass and energy.

Hawking radiation.

The animation at the top cartoons the Hawking radiation from a black hole of the size shown at the bottom. The blobs are supposed to be individual photons. Notice, first, that the photons have `sizes' (wavelengths) comparable to the size of the black hole, and, second, that the Hawking radiation is not very bright - the black hole emits roughly one photon every light crossing time of the black hole.



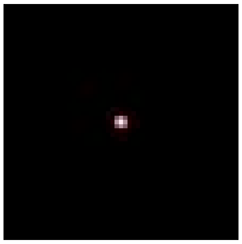
Andrew J. S. Hamilton, Box 440, JILA
University of Colorado, Boulder, Colorado 80309

Black hole evaporation.

- Hawking radiation is emitted more efficiently if the gravity at the horizon is stronger (i.e. its temperature is higher).
  - Recall: horizon gravity is stronger for smaller-mass black holes.
- Thus an isolated black hole will eventually evaporate, as it radiates away all of its mass-energy. The smaller the black hole mass is, the larger the evaporation rate is.
- The time it takes to evaporate:
  - 10^9 M_{sun} black hole: 10^{94} years.
  - 2 M_{sun} black hole: 10^{67} years.
  - 10^8 gram black hole: 1 second (!)

Black hole evaporation.

It is not well established what an evaporating mini black hole would actually look like in realistic detail. The Hawking radiation itself would consist of fiercely energetic particles, antiparticles, and gamma rays. Such radiation is invisible to the human eye, so optically the evaporating black hole might look like a dud. However, it is also possible that the Hawking radiation, rather than emerging directly, might power a hadronic fireball that would degrade the radiation into particles and gamma rays of less extreme energy.



Andrew J. S. Hamilton, Box 440, JILA
University of Colorado, Boulder, Colorado 80309

Hawking radiation.

- Hawking radiation is really tiny and usually insignificant compared to other sources of light commonly found near black holes.
- That's why it takes black holes so long to evaporate. Consider how it works out for a 2 M_{sun} black hole.
- Luminosity, if accreting at its maximum (Eddington) rate:

$$L = 25\times10^{38} \text{ erg/s} = 6.6\times10^{4} L_{sun}$$

- Luminosity due to Hawking radiation:

$$L = 2.3\times10^{-22} \text{ erg/s} = 5.9\times10^{-56} L_{sun}$$

Exotic matter.

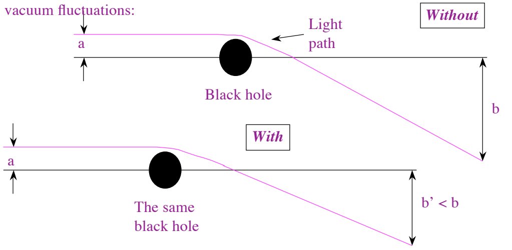
- The foregoing should strike you as weird. Why is it that the black hole can *consume* a particle, and wind up *decreasing* in mass and energy?
  - Because in the strongly warped space-time near the horizon, virtual particles made from vacuum fluctuations turn out to have negative energy **density**.
    - Energy density = energy per unit volume.
  - These particles indeed have positive mass -- look at the one that escaped! -- but their mass is distributed very strangely over space-time. (Quantum-mechanically speaking, particles have nonzero volume; this is an aspect of the wave-particle duality.)
  - Matter with negative energy density is generally called **exotic matter.**

Exotic matter.

- Theoretical details of exotic matter, according to the present partial marriage of general relativity and quantum mechanics (incompletely known; only studied intensively since 1985):
  - Quantum mechanical vacuum fluctuations in flat space-time - far from any strong gravitational field - always have zero net energy density; they can never be exotic.
  - However, in warped space-time, vacuum fluctuations are in general exotic: their net energy density is negative, according to a distant observer measuring the energy density by observation of the deflection of light by the ensemble of fluctuations. The stronger the curvature, the more negative the energy density looks.

Exotic matter.

Results of **calculations** of deflection of light by a black hole, with and without vacuum fluctuations:



Light is deflected less when vacuum fluctuations are included in the calculations: thus these fluctuations are "anti-gravity" (i.e. exotic).

That’s all for today! Next week we will explore what it is like inside a black hole.

The Peekskill meteor of 1992.