Physics 235, Midterm Exam # 2

November 16, 2004: 9.40 am - 10.55 am

Instructions:

- Do not turn this page until you are instructed to do so.
- Each answer must be well motivated. You will need to show your work in order to get full credit.
- If we can not read what you wrote, we can only assume it is wrong.
- Useful equations are attached to the exam and a copy of Appendices D, E, and F is available as a separate handout.
- The total number of points you can get on this exam is 105 (you get 5 bonus point if you answer question 5 correctly).

Problem 1 (25 points)

A bucket of water is set spinning around its vertical symmetry axis with a constant angular velocity ω.

a. Consider a small volume of water of mass m, located on the surface, a distance r from the symmetry axis. What is the effective force on this mass of water?
b. After a while, the shape of the surface of the water in the bucket does not change anymore. What is the effective force on the small volume of water at that time?
c. What is the pressure gradient acting on this volume of water? You can express your answer in vector notation.
d. When the system has reached a state of equilibrium, what is the function $z(r)$ that describes the shape of the surface (z is the vertical displacement and r is the distance from the rotation axis)?

Problem 2 (25 points)

Consider a simple plane pendulum consisting of a mass m attached to a massless string of length l. After the pendulum is set into motion, the length of the string is shortened at a constant rate:

$$\frac{dl}{dt} = -\alpha = \text{constant}$$

The suspension point remains fixed.

a. Compute the Lagrangian function.
b. Compute the Hamiltonian function.
c. Compare the Hamiltonian and the total energy, and discuss the conservation of energy for the system.

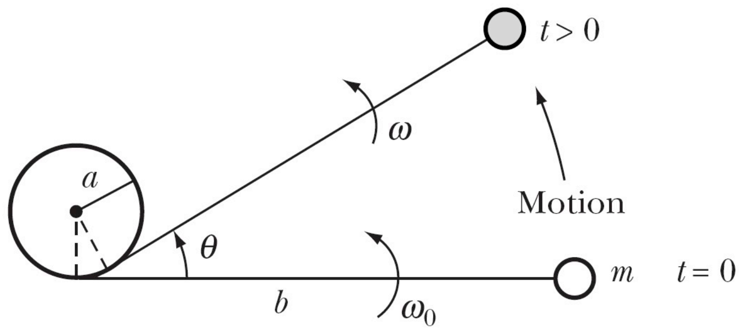
Problem 3 (25 points)

Two double stars, having a mass M_{sun} and $3M_{sun}$, rotate in circular orbits about their common center of mass. Their separation is L.

a. What is the radius of the circular orbit of the lighter star?
b. What is the orbital velocity of the lighter star?
c. What is the period of rotation of the lighter star?
d. What is the period of rotation of the heavier star?

Problem 4 (25 points)

A particle of mass m at the end of a massless cord wraps itself around a vertical cylinder of radius a. All the motion is in the horizontal plane, and is not influenced by gravity. The angular velocity of the cord is ω_0 when the distance from the point of contact of the cord and the cylinder to the particle is b.



a. Is kinetic energy conserved? If so, why? If not, why not?
b. Is the angular momentum of the mass about the center of the cylinder conserved? If so, why? If not, why not?
c. What is the angular velocity of the mass after the cord has turned through an additional angle θ?
d. What is the tension in the cord after the cord has turned through an additional angle θ?

Problem 5 (5 points)

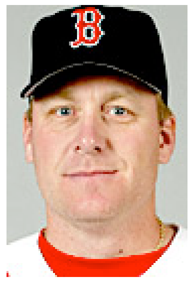
a. When is the next time I expect the Red Sox to win a world series?

___ 2190
___ 2090
___ 2005
___ never

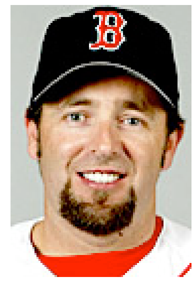
b. Match the pictures to the names listed below.


A


B


C


D


E

___ Curt Schilling
___ Manny Ramirez
___ David Ortiz
___ Kevin Millar
___ Johnny Damon

Useful Relations

$$\bar{A} \bullet \bar{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\bar{C} = \bar{A} \times \bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\bar{A} \bullet \left(\bar{B} \times \bar{C} \right) = \bar{B} \bullet \left(\bar{C} \times \bar{A} \right) = \bar{C} \bullet \left(\bar{A} \times \bar{B} \right)$$

$$\bar{A} \times \left(\bar{B} \times \bar{C} \right) = \left(\bar{A} \bullet \bar{C} \right) \bar{B} - \left(\bar{A} \bullet \bar{B} \right) \bar{C}$$

$$\left(\bar{A} \times \bar{B} \right) \bullet \left(\bar{C} \times \bar{D} \right) = \bar{A} \bullet \left[\bar{B} \times \left(\bar{C} \times \bar{D} \right) \right]$$

$$\left(\bar{A} \times \bar{B} \right) \times \left(\bar{C} \times \bar{D} \right) = \left[\left(\bar{A} \times \bar{B} \right) \bullet \bar{D} \right] \bar{C} - \left[\left(\bar{A} \times \bar{B} \right) \bullet \bar{C} \right] \bar{D}$$

Euler's equations without external constraints:

$$\frac{\partial f}{\partial y_i} - \frac{d}{dx} \left(\frac{\partial f}{\partial y_i'} \right) = 0$$

Euler's equations with external constraints $g\{y; x\} = 0$:

$$\left(\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) \right) + \lambda(x) \left(\frac{\partial g}{\partial y} \right) = 0$$

where $\lambda(x)$ is the Lagrange undetermined multiplier.

Hamilton's principle:

$$\delta \int_{t_1}^{t_2} \left(T - U \right) dt = 0$$

The Lagrange equations of motion:

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} = 0$$

Lagrange equations of motion with undetermined multipliers:

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \sum_{k=1}^{m} \lambda_k(t)\frac{\partial f_k}{\partial q_j} = 0$$

Definition of the Hamiltonian:

$$H = \sum_j \left(\dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) - L$$

Hamilton's equations of motion:

$$\frac{\partial H}{\partial p_j} - \dot{q}_j = 0$$

$$\frac{\partial H}{\partial q_j} + \dot{p}_j = 0$$

Orbital motion:

$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{l^2}{\mu r^2} + U(r)$$

$$\frac{dr}{dt} = \pm\sqrt{\frac{2}{\mu}(E-U) - \frac{l^2}{\mu^2 r^2}}$$

$$\theta(r) = \pm\int \frac{\left(\frac{l}{\mu r^2}\right)}{\sqrt{\frac{2}{\mu}(E-U) - \frac{l^2}{\mu^2 r^2}}}\,dr$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{l^2}F(r)$$

Effective potential energy:

$$V(r) = U(r) + \frac{l^2}{2\mu r^2}$$

Center of Mass:

$$\bar{R}_{cm} = \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i} = \frac{1}{M} \sum_i m_i \bar{r}_i$$

Linear momentum of a system of particles:

$$\bar{P} = M\dot{\bar{R}}$$

$$\frac{d\bar{P}}{dt} = \bar{F}_{ext}$$

Angular momentum of a system of particles:

$$\bar{L} = \bar{L}_{cm} + \bar{L}_{wrt,cm}$$

$$\frac{d\bar{L}}{dt} = \sum_i \left\{ \bar{r}_i \times \bar{F}_{i,ext} \right\}$$

Scattering cross sections:

$$\sigma(\theta) = \frac{\text{\#of interactions per target nucleus into an area d}\Omega'\text{ at }\theta}{\text{\# of incident particles per units area}} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Rutherford scattering:

$$\sigma(\theta) = \left(\frac{k}{4T_0'}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Rocket equations:

$$\mathrm{R\,u = M\,a}$$

$$\mathrm{v_f = v_i + u \ln\left(\frac{M_i}{M_f}\right)}$$

Fixed and rotating coordinate systems:

$$v_f = \left(\frac{d\bar{r}\,'}{dt} \right)_{fixed} = \left(\frac{d\bar{R}}{dt} \right)_{fixed} + \left(\frac{d\bar{r}}{dt} \right)_{rotating} + \bar{\omega} \times \bar{r} = V + v_r + \bar{\omega} \times \bar{r}$$

Effective forces in rotating coordinate systems:

$$\bar{F}_{eff} = m\bar{a}_f - 2m\bar{\omega} \times \bar{v}_r - m\bar{\omega} \times \left\{ \bar{\omega} \times \bar{r} \right\}$$