Classical Mechanics
Phy 235, Lecture 21.

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Course Announcements.

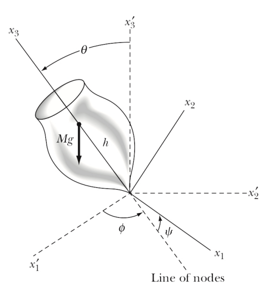
- Exam # 3 will take place on December 4 between 8 am and 9.30 am in B&L 109. The material covered on the exam will be reviewed on Monday December 3.
- Homework set # 10 (the last homework assignment) is due on Friday November 30 at noon.
- **The term paper for this course is due on Sunday, December 9, at noon.**
- Today we will finish the discussion of Chapter 11 and start the discussion of Chapter 12.

Symmetric top with external torque.

- For this top, $I_1 = I_2$.
- There is a non-zero torque along the x_1 and x_2 axes. **Note: the lecture note on page 21 do not include the torque along these two axes.**
- We conclude:

$$(I_1 - I_2)\omega_1\omega_2 - I_3\dot{\omega}_3 = -I_3\dot{\omega}_3 = 0$$

$$\omega_3 = \text{constant}$$



Symmetric top with external torque. Effective energy.

- Since no non-conservative forces are acting on the system, energy is conserved.
- Sine the angular velocity around the $x3$ axis is constant, we can subtract it from the total energy. This produces the ***effective energy E'***:

$$E' = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} \frac{\left(p_\phi - p_\psi \cos\theta\right)^2}{I_1 \sin^2\theta} + Mgh\cos\theta$$

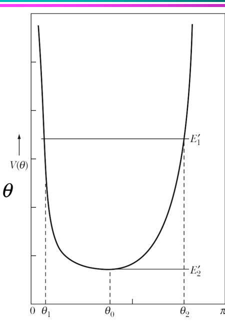
- The momenta are constant and the effective energy only depends on θ and $d\theta/dt$. The problem has been reduced to a one-dimensional problem.

Symmetric top with external torque. Effective potential energy.

- One term of the effective energy depends on $d\theta/dt$, the last two terms depend on θ.
- The effective potential energy is defined as

$$V(\theta) = \frac{1}{2} \frac{\left(p_\phi - p_\psi \cos\theta\right)^2}{I_1 \sin^2\theta} + Mgh\cos\theta$$

- The potential energy shows the limits of motion for a given effective energy.



Symmetric top with external torque.

- The effective energy has a minimum value at a specific angle θ_0.
- At this angle stable precession can be produced if the angular velocity is sufficiently large.

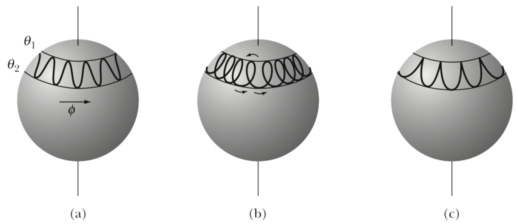
$$\omega_3 \geq \frac{2}{I_3}\sqrt{MghI_1\cos\theta_0}$$

- In general there are two precession rates since the there are two possible values of β:

$$\dot{\phi} = \frac{p_\phi - p_\psi \cos\theta_0}{I_1 \sin^2\theta_0} = \frac{\beta}{I_1 \sin^2\theta_0}$$

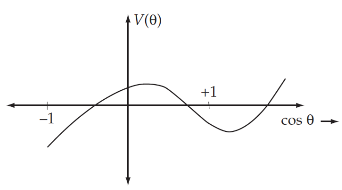
Symmetric top with external torque.

- When the angle of inclination is not θ_0, the system will nutate.



Problem 11.30.

Investigate the equation for the turning points of the nutational motion by setting $d\theta/dt = 0$ in the equation of the effective energy. Show that the resulting equation is cubic in $\cos\theta$ and has two real roots and one imaginary root.



Stability of Rigid-Body Rotations.

- The rotation of a rigid body is stable if the system, when perturbed from its equilibrium condition, carries out small oscillations about it.
- Consider we use the principal axes of rotation to describe the motion, and we choose these axes such that $I_3 > I_2 > I_1$:
  - If the system rotates around the x_1 axis, small perturbations around the x_2 and x_3 axes will cause it to oscillate around the equilibrium values. Rotation around the x_1 axis is stable.
  - Rotation around the x_2 axis is unstable.
  - Rotation around the x_3 axis is unstable.

Problem 11.34.

Consider a symmetrical rigid body rotating freely about its center of mass. A frictional torque ($N_f = -b\omega$) acts to slow down the rotation. Find the component of the angular velocity along the symmetry axis as a function of time.

4 Minute 7 Second Intermission.

- Since paying attention for 1 hour and 15 minutes is hard when the topic is physics, let's take a 4 minute 7 second intermission.
- You can:
  - Stretch out.
  - Talk to your neighbors.
  - Ask me a quick question.
  - Enjoy the fantastic music.

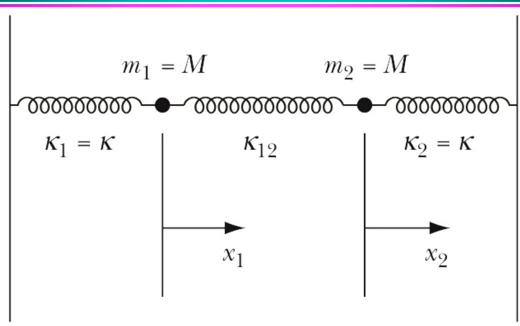
And now for something completely different:

CHAPTER 12
COUPLED OSCILLATIONS

Coupled Oscillations

- Coupled oscillators:
  - Oscillators that are connected in such a way that energy can be transferred between them.
  - The motion of coupled oscillators is complex and in general not periodic.
  - We can always find a coordinate frame in which each oscillator oscillated with a well-defined frequency.

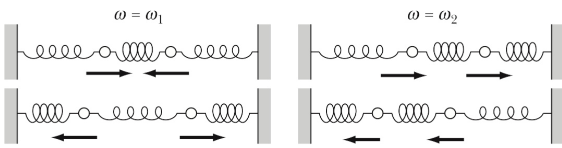
Two Coupled Harmonic Oscillators.



Two Coupled Harmonic Oscillators.

- Two approaches:
- Approach 1:
  - Write down the coupled equations of motion.
  - Try trial functions for x_1 and x_2 with the same frequency.
  - The two frequency will have different amplitudes.
- Approach 2:
  - Cary out a coordinate transformation to decouple the coupled equations.
  - Solve each decoupled equation.
  - Each solution may have a different frequency.
  - Use the solutions of the decoupled equations and the “inverse” coordinate transformation to find the solution.

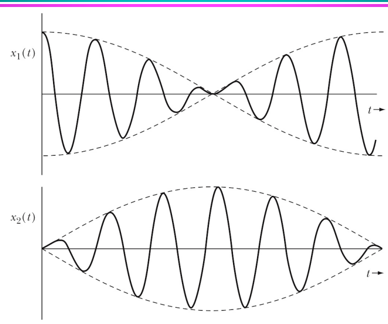
Two Coupled Harmonic Oscillators. Two modes.



Antisymmetrical mode (out of phase)

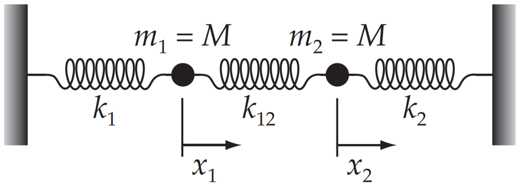
Symmetrical mode (in phase)

Weak Coupling



Problem 12.1.

Reconsider the problem of two coupled oscillators discussion in Section 12.2 in the event that the three springs all have different force constants. Find the two characteristic frequencies, and compare the magnitudes with the natural frequencies of the two oscillators in the absence of coupling.



Problem 12.3.

Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such that by adding to the system a mass m, common to both oscillators, the equations of motion become

$$\left.\begin{aligned} \ddot{x}_1 + \frac{m}{M}\ddot{x}_2 + \omega_0^2 x_1 = 0 \\ \ddot{x}_2 + \frac{m}{M}\ddot{x}_1 + \omega_0^2 x_2 = 0 \end{aligned}\right]$$

Solve this pair of coupled equations, and obtain the frequencies of the normal modes of the system.

ENOUGH FOR TODAY?