Physics 237, Midterm Exam #2

Tuesday March 27, 2012
8.00 am – 9.30 am

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, nor each other.

1. Problems 1 and 2 must be answered in booklet # 1.

2. Problems 3 and 4 must be answered in booklet # 2.

3. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, your equation sheet, and the two blue exam booklets. **All** items must be clearly labeled with your name, your student ID number, and the day/time of your workshop.

Name: _____

ID number: _____

Workshop Day/Time: _____

One-Electron Atoms – Details

The following table lists the $n = 1$, $n = 2$, and $n = 3$ wavefunctions of the one-electron atom.

Table 7-2 Some Eigenfunctions for the One-Electron Atom

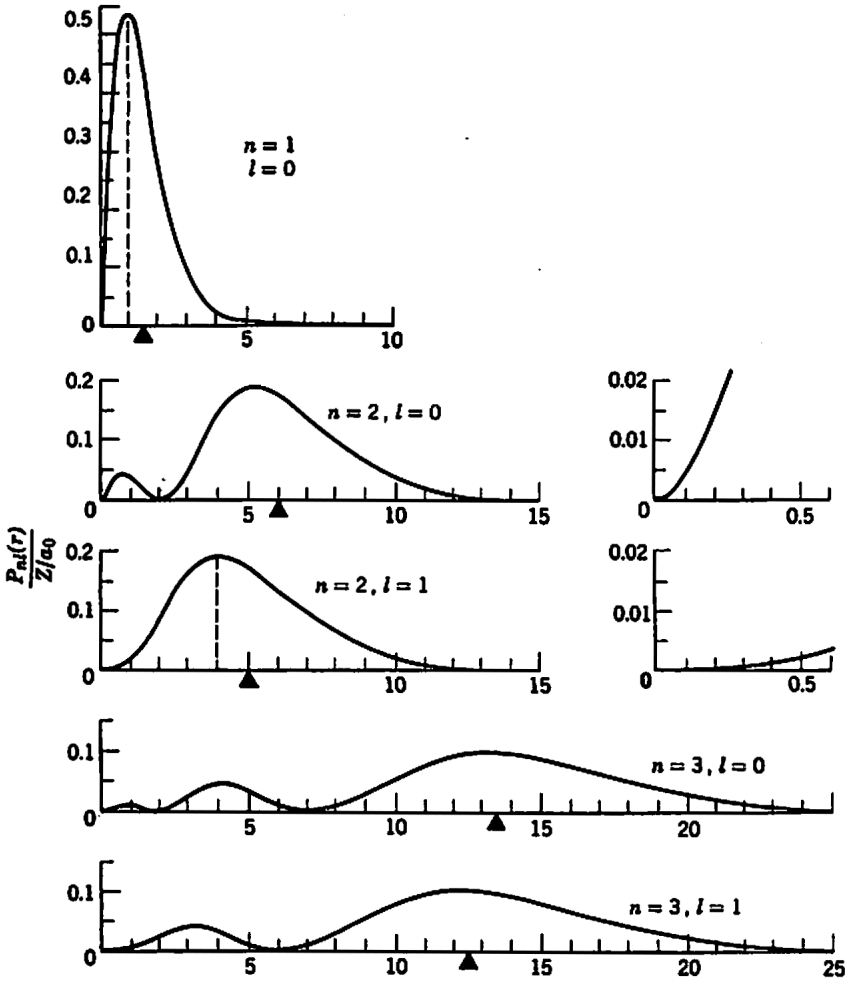
Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2 - \frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0}e^{-Zr/2a_0}\cos\theta$
2	1	± 1	$\psi_{21\pm 1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0}e^{-Zr/2a_0}\sin\theta\, e^{\pm i\varphi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(27 - 18\frac{Zr}{a_0} + 2\frac{Z^2r^2}{a_0^2}\right)e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0}e^{-Zr/3a_0}\cos\theta$
3	1	± 1	$\psi_{31\pm 1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0}e^{-Zr/3a_0}\sin\theta\, e^{\pm i\varphi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}(3\cos^2\theta - 1)$
3	2	± 1	$\psi_{32\pm 1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}\sin\theta\cos\theta\, e^{\pm i\varphi}$
3	2	± 2	$\psi_{32\pm 2} = \frac{1}{162\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}\sin^2\theta\, e^{\pm 2i\varphi}$

In these wavefunctions, the parameter a_0 is defined as

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

The energy of each wavefunction is equal to

$$E_n = -\frac{\mu Z^2 e^4}{\left(4\pi\varepsilon_0\right)^2 2\hbar^2 n^2}$$



The radial probability density for the electron in a one-electron atom for n = 1, 2, 3 and various values of l.

Problem 1 (30 points) **ANSWER IN BOOKLET 1**

Consider the ground-state wavefunctions of the Hydrogen atom (see Page 2).

a) What is the expectation value $\langle V \rangle$ of the potential energy of the hydrogen atom when it is in its ground state?

b) Express the energy of the ground state of the hydrogen atom in terms of the expectation value $\langle V \rangle$ of the potential energy.

c) What is the expectation value of the kinetic energy of the ground state?

Problem 2 (30 points) **ANSWER IN BOOKLET 1**

Consider the following eigenfunction of the electron in a one-electron atom with atomic number *Z*:

$$\psi = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/(2a_0)} \sin\theta \, e^{+i\varphi}.$$

a) What is the magnetic quantum number m_ℓ of this eigenfunction?
Note: you will not receive any credit for a correct answer if it is not properly justified. You cannot justify your answer on the basis of the information provided on page 2 of the exam.

b) What is the azimuthal quantum number ℓ of this eigenfunctions?
Note: you will not receive any credit for a correct answer if it is not properly justified. You cannot justify your answer on the basis of the information provided on page 2 of the exam.

c) How would you calculate the energy of the electron?
Note: you do not have to do the actual calculation, but you should provide sufficient details on how you would carry out this calculation.

Problem 3 (40 points) **ANSWER IN BOOKLET 2**

A particle of total energy $9V_0$ is incident from the left on a potential given by

$$V = \begin{cases} 8V_0 & x < 0 \\ 0 & 0 < x < a \\ 5V_0 & a < x \end{cases}$$

Assume the general solution of the wavefunction of this particle has the following form:

$$\psi(x) = \begin{cases} A\, f_1(x) + B\, g_1(x) & x < 0 \\ C\, f_2(x) + D\, g_2(x) & 0 < x < a \\ E\, f_3(x) & a < x \end{cases}$$

a) Write down the general solution for the wavefunction in the $x < 0$ region; that is, determine the functions $f_1(x)$ and $g_1(x)$. Note: you do not yet have to determine the value of the constants A and B that appear in the wavefunction.

b) Write down the general solution for the wavefunction in the $0 < x < a$ region; that is, determine the functions $f_2(x)$ and $g_2(x)$. Note: you do not yet have to determine the value of the constants C and D that appear in the wavefunction.

c) Write down the general solution for the wavefunction in the $a < x$ region; that is, determine the function $f_3(x)$. Note: you do not yet have to determine the value of the constant E that appears in the wavefunction.

d) Assume that $A = 1$. Use the boundary conditions at $x = 0$ to determine a relation between the constants C and D.

e) Use the boundary conditions at $x = a$ to determine the constant E.

f) What is the probability that the particle will be transmitted to the positive side of the potential well ($x > a$)? **Note: you can express your answer in terms of the constant *E*.**

Problem 4 (5 points) **ANSWER IN BOOKLET 2**

Which of the following sports team is the favorite Dutch team of your Phy 237 instructor?

a) The Yankees
b) The Buffalo Bills
c) The Red Sox
d) AJAX

Please enter your answer in your exam booklet.