Physics 237, Midterm Exam #3

Tuesday April 24, 2012

8.00 am – 9.30 am

Do not turn the pages of the exam until you are instructed to do so.

Exam rules: You may use *only* a writing instrument while taking this test. You may *not* consult any calculators, computers, books, nor each other.

1. Problems 1 and 2 must be answered in booklet # 1.

2. Problems 3 and 4 must be answered in booklet # 2.

3. The answers need to be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. Answers that are not motivated will not receive any credit, even if correct.

At the end of the exam, you need to hand in your exam, your equation sheet, and the two blue exam booklets. **All** items must be clearly labeled with your name, your student ID number, and the day/time of your workshop.

Name: ____

ID number: ____

Workshop Day/Time: ___________________________________

One-Electron Atoms – Details

The following table lists the $n = 1$, $n = 2$, and $n = 3$ wavefunctions of the one-electron atom.

Table 7-2 Some Eigenfunctions for the One-Electron Atom

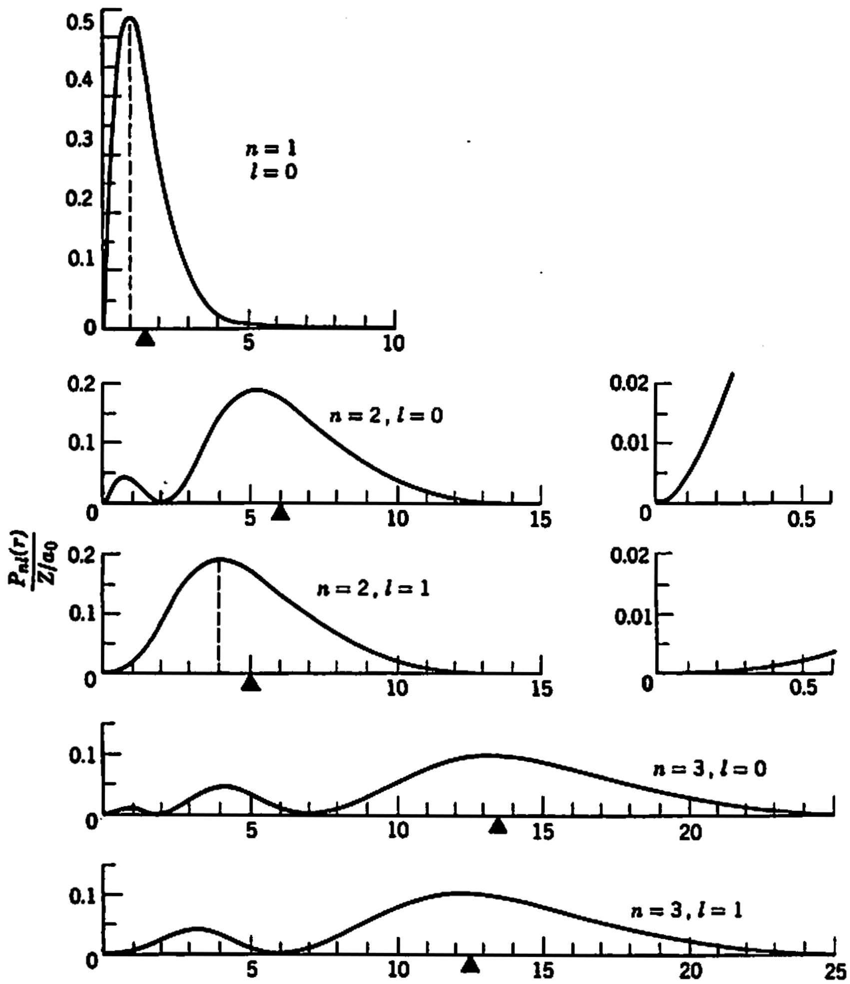
Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2 - \frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\cos\theta$
2	1	±1	$\psi_{21\pm1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\sin\theta\, e^{\pm i\varphi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(27 - 18\frac{Zr}{a_0} + 2\frac{Z^2r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\cos\theta$
3	1	±1	$\psi_{31\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6 - \frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\sin\theta\, e^{\pm i\varphi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}(3\cos^2\theta - 1)$
3	2	±1	$\psi_{32\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin\theta\cos\theta\, e^{\pm i\varphi}$
3	2	±2	$\psi_{32\pm2} = \frac{1}{162\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin^2\theta\, e^{\pm 2i\varphi}$

In these wavefunctions, the parameter a_0 is defined as

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}$$

The energy of each wavefunction is equal to

$$E_n = -\frac{\mu Z^2 e^4}{\left(4\pi\varepsilon_0\right)^2 2\hbar^2 n^2}$$



The radial probability density for the electron in a one-electron atom for n = 1, 2, 3 and various values of l.

Problem 1 (30 points) **ANSWER IN BOOKLET 1**

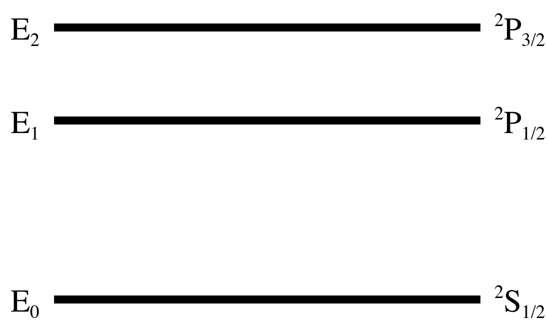
a) Prove that any two different non-degenerate bound wavefunctions $\psi_i(x)$ and $\psi_j(x)$ that are solutions to the time-independent Schrödinger equation for the same potential $V(x)$ obey the orthogonality relation

$$\int_{-\infty}^{\infty} \psi_j^*(x)\psi_i(x)\,dx = 0 .$$

b) What changes in part a) when the wavefunctions $\psi_i(x)$ and $\psi_j(x)$ are degenerate?

Problem 2 (35 points) **ANSWER IN BOOKLET 1**

Consider the three lowest energy levels in Na, shown in the Figure below.



a) What are the Landé *g* factors for these levels?

b) When the atom is placed in a weak magnetic field, the energy levels split. Draw an energy level diagram showing the energy levels and determine the corresponding energies (in terms of E_0, E_1, and E_2).

c) Which transitions between the ^2P and the ^2S energy levels are possible? Explain why you selected these transitions?

Problem 3 (30 points) **ANSWER IN BOOKLET 2**

N distinguishable atoms are distributed over two energy levels $E = 0$ and $E = E_1$ in a system that is maintained at temperature T.

a) What is the energy of the system?

b) What is the heat capacity of the system?

Problem 4 (5 points) **ANSWER IN BOOKLET 2**

Who first used the following quote:

“ *May the odds be ever in your favor!* “

a) Derek Jeter
b) Katniss Everdeen
c) Harry Potter
d) Frank Wolfs
e) Effie Trinket
f) Joe Girardi