Physics 121, Final Exam

Do not turn the pages of the exam until you are instructed to do so.

You are responsible for reading the following rules carefully before beginning.

Exam rules: You may use only a writing instrument while taking this test.

You may not consult any calculators, computers, books, notes, or each other.

Procedure:

1. Answer the multiple-choice questions (problems 1 – 10) by marking your answer on the scantron form. For each multiple-choice question (problems 1 – 10), select only one answer. **Questions with more than one answer selected will be considered incorrect.**

2. The analytical problems (11 – 18) must be answered in the blue exam booklets. You must answer problems 11, 12, and 13 in booklet 1, problems 14, 15, and 16 in booklet 2, and problems 17 and 18 in booklet 3. **If you do not follow this convention there is no guarantee that the problems that appear in the wrong booklet will be graded.**

3. The answer to each analytical problem must be well motivated and expressed in terms of the variables used in the problem. You will receive partial credit where appropriate, but only when we can read your solution. **Answers that are not motivated will not receive any credit, even if correct.**

4. At the end of the exam, you must hand in the blue exam booklets, the scantron form, the exam, and the formula sheets. All items must be clearly labeled with your name and student ID number. **If any of these items is missing, we will not grade your exam, and you will receive a score of 0 points.**

Note: If you do not answer a question in terms of the variables provided, you will not receive credit for that question.

Note: Your grade on the final exam will be based on the numerical sum of the score for the multiple-choice questions (25 points) and the sum of the scores of 7 best analytical questions (we will drop the worst result on the analytical questions from the final exam grade).
Problem 1 (2.5 points)

Two blocks of the same size but different masses, \( m_1 \) and \( m_2 \), are placed on a table side-by-side in contact with each other. Assume that \( m_1 > m_2 \). Let \( N_1 \) be the normal force between the two blocks when you push horizontally on the free side of \( m_1 \) (towards \( m_2 \)). Let \( N_2 \) be the normal force between the two blocks when you push horizontally on the free side of \( m_2 \) (towards \( m_1 \)). Which of the following statements is true?

1. \( N_1 = N_2 \)
2. \( N_1 < N_2 \)
3. \( N_1 > N_2 \)

Problem 2 (2.5 points)

A ball slides down an inclined track and then rounds a loop-the-loop. The ball is released from an initial height so that it has just enough speed to go around the loop without falling off. At the top of the loop-the-loop, the normal force of the loop on the ball is

1. equal to the weight of the ball and pointing down.
2. equal to the weight of the ball and pointing up.
3. equal to twice the weight of the ball and point up.
4. equal to zero.

Problem 3 (2.5 points)

What is the force that corresponds to the potential energy function \( U(x, y) = 3xy + 5x^2 + 6y^3 \)?

1. \( \mathbf{F} = 5x^2 \mathbf{i} + 6y^3 \mathbf{j} \)
2. \( \mathbf{F} = 3xy \mathbf{i} + 3xy \mathbf{j} \)
3. \( \mathbf{F} = (-3y - 10x) \mathbf{i} + (-3x - 18y^2) \mathbf{j} \)
4. \( \mathbf{F} = (3y + 10x) \mathbf{i} + (-3x - 18y^2) \mathbf{j} \)
Problem 4 (2.5 points)

Three uniform spheres of radii $2R$, $R$, and $3R$ are placed in contact next to each other on the $x$ axis in this order (the smallest sphere is in the center, the $2R$ sphere is located to the left, and the $3R$ sphere is located to the right). The centers of the spheres are located on the $x$ axis. What is the distance from the center of mass of this system from the center of the smallest sphere, assuming that each sphere has the same density?

1. $(7/3)R$
2. $(1/3)R$
3. $(3/7)R$
4. $(65/36)R$

Problem 5 (2.5 points)

The precession rate of the a spinning top

1. is proportional to its angular momentum
2. does not depend upon its angular momentum
3. is inversely proportional to its angular momentum
4. is inversely proportional to its kinetic energy

Problem 6 (2.5 points)

For most materials, how is the coefficient of volume expansion related to the coefficient of linear expansion?

1. The coefficient of volume expansion is equal to the coefficient of linear expansion.
2. The coefficient of volume expansion is one-third the coefficient of linear expansion.
3. The coefficient of volume expansion is three times the coefficient of linear expansion.
4. The coefficient of volume expansion is twice the coefficient of linear expansion.
Problem 7 (2.5 points)

Match the above shown players with the following names:

A. Manny Ramirez  
B. Jason Varitek  
C. Daisuke Matsuzaka  
D. David Ortiz

1234 =

1. ABCD  
2. ACDB  
3. BADC  
4. BDAC  
5. CADB  
6. CABD  
7. DBAC  
8. DCBA
Problem 8 (2.5 points)

Consider the following graph, showing position versus time for simple harmonic motion.

What is the frequency of this motion?

1. 0.25 Hz
2. 0.50 Hz
3. 1.0 Hz
4. 4.0 Hz

Problem 9 (2.5 points)

By what factor does the RMS speed of an ideal gas change when the absolute temperature of the gas is doubled?

1. 2
2. 4
3. $\frac{1}{2}$
4. $\sqrt{2}$
Problem 10 (2.5 points)

A system consisting of a fixed amount of gas starts at pressure $P_1$ and volume $V_1$ and ends up at pressure $P_2$ and volume $V_2$ after some thermodynamic process. Which of the following quantities do not depend on the path taken on a pressure–volume diagram during the process?

1. Work done on the environment by the system during the process.
2. Work done by the environment on the system during the process.
3. The heat added to the system during the process.
4. The final temperature of the system.
Problem 11 (25 points)  

The operation of an automobile internal combustion engine can be approximated by a reversible cycle known as the Otto cycle, whose $PV$ diagram is shown in the Figure below. The gas in cylinder at point a is compressed adiabatically to point b. Between point b and point c, heat is added to the gas, and the pressure increases at constant volume. During the power stroke, between point c and point d, the gas expands adiabatically. Between point d and point a, heat is removed from the system, and the pressure decreases at constant volume. Assume the gas is an ideal monatomic gas.

(a) Assuming there are $n$ moles of gas in system, what are the heats $|Q_H|$ and $|Q_L|$? Express your answer in terms of $n, R, T_a, T_b, T_c,$ and $T_d$.

(b) What is the efficiency of the Otto cycle? Express your answer in terms of $T_a, T_b, T_c,$ and $T_d$.

(c) Express the efficiency of the Otto cycle in terms of just the compression ratio $V_a/V_b$ and $\gamma$.

   Hint: use the fact that during an adiabatic process $PV^\gamma = \text{constant}$.

(d) How does the efficiency change when we replace the monatomic gas with a diatomic gas?
Problem 12 (25 points)  

A tunnel is bored through the Earth along a diameter, as shown in the Figure below. Assume that the earth is a homogeneous sphere with total mass $M$ and radius $R$.

(a) A package with mass $m$ is dropped into the tunnel. Use the shell theorem to calculate the gravitational force acting on the package as function of the distance $r$ from the center of the Earth.

(b) Show that the package will oscillate back and forth with simple harmonic motion.

(c) If the tunnel were used to deliver mail, how long would it take for a letter to travel through the Earth?

Express all your answer in terms of $G$, $m$, $M$, $r$, and $R$. 
Problem 13 (25 points)  
Answer in Exam Booklet 1

A thin horizontal bar AB of mass \( m \) and length \( L \) is pinned to a vertical wall at A and supported at B by a thin wire BC that makes an angle \( \theta \) with the horizontal. A block with mass \( M \) can be moved anywhere along the bar. The distance \( x \) is defined as the distance between the center of mass of the block and the wall (see Figure). The system is in equilibrium.

(a) What is the tension in the thin wire as function of \( x \)?

(b) What is the horizontal component of the force exerted on the bar by the pin at A as function of \( x \)?

(c) What is the vertical component of the force exerted on the bar by the pin at A as function of \( x \)?

Express all you answers in terms of \( m, M, x, L, \theta, \) and \( g \).
Problem 14 (25 points)

A bullet of mass $m$ is fired horizontally at two blocks resting on a smooth table top, as shown in the Figure. The bullet passes through the first block of mass $M_1$, and embeds itself in a second block of mass $M_2$. Speeds equal to $v_1$ and $v_2$, respectively, are thereby imparted on the blocks, as shown in the Figure. The mass removed from the first block by the bullet can be neglected.

**Answer in Exam Booklet 2**

(a) What is the speed of the bullet immediately after emerging from the first block?

(b) What is the original speed of the bullet?

Express all you answers in terms of $M_1, M_2, v_1, v_2, m$. 
Problem 15 (25 points)

(a) A car of mass \( m \) moves at a constant speed \( v_0 \) on a curved unbanked roadway whose radius of curvature is equal to \( R \). The car is able to make the turn without skidding of the road. What must be the minimum coefficient of static friction \( \mu_s \) between the tires and the roadway?

(b) You cannot count on a sideways frictional force to get your car around a curve if the road is icy or wet. That is why highways are banked. Suppose a car of mass \( m \) moves at a constant speed \( v \) on a curved banked highway whose radius of curvature is equal to \( R \) and whose angle of bank is equal to \( \theta \). There is no friction between the tires and the roadway. What speed must the car have in order to round the curve without skidding of the road?

Express all your answers in terms of \( m, R, \theta, v_0, \) and \( g \).
Problem 16 (25 points)  
A hoop of mass $M$ and radius $R$ is rolling down a ramp that slopes at an angle $\theta$ (see Figure). The hoop starts from rest, a distance $L$ up the slope, and rolls without slipping. The entire mass of the hoop is distributed uniformly along its rim.

(a) What is the friction force $f$ between the hoop and the surface of the slope?
(b) What is the linear acceleration of the rolling hoop?
(c) How long does it take the hoop to reach the bottom of the ramp?
(d) What is the velocity of the hoop at the bottom of the ramp?
(e) What happens if there is no friction between the ramp and the hoop?

Express all your answers in terms of $M$, $R$, $\theta$, $L$, and $g$. 
Problem 17 (25 points)

Consider a composite slab of material of cross-sectional area $A$, consisting of two materials having different thicknesses, $L_1$ and $L_2$, and different thermal conductivities, $k_1$ and $k_2$. The temperatures of their outer surfaces are $T_h$ and $T_c$.

![Diagram of composite slab with temperatures $T_h$, $T_c$, and layers $L_1$, $L_2$]

a) What is the temperature $T_x$ at the interface between the two materials of the slab?

b) Calculate the rate of heat transfer through this composite slab.

Express all your answers in terms of $A$, $L_1$, $L_2$, $k_1$, $k_2$, $T_h$, and $T_c$. 

Problem 18 (25 points)  

A rigid rod of mass $M$ and length $L$ rotates in a vertical plane about a frictionless pivot through its center (see Figure). Two solid spheres of masses $m_1$ and $m_2$ are attached to the ends of the rod. Assume $m_1 > m_2$.

a) Determine the angular momentum of the system when the angular velocity is equal to $\omega$.

b) Determine the magnitude of the angular acceleration of the system when the rod makes an angle $\theta$ with the horizontal.

Express all your answers in terms of $m_1, m_2, M, L, \theta,$ and $\omega$. 