Problem 1 (35 points)
Check Stoke's theorem for the vector function
\[ \vec{v}(x,y,z) = y \hat{k} \]
using the triangular surface shown in Figure 1. Express all your answers in terms of \( a \).

![Figure 1. Problem 1.](image)

Problem 2 (30 points)
A spherical shell of radius \( R \) has a uniform surface charge density. The total charge on the shell is \( q \). Find the electrostatic energy of this system in two different ways. Express all your answers in terms of \( q \) and \( R \).

Problem 3 (35 points)
Consider the following three vector functions:

1. \[ \vec{E}_1(x,y,z) = \alpha \left[ (y^2) \hat{i} + (2xy + z^2) \hat{j} + (2yz) \hat{k} \right] \]
2. \[ \vec{E}_2(x,y,z) = \alpha \left[ (y^2) \hat{i} + (2xy + z^2) \hat{j} - (2yz) \hat{k} \right] \]
\[ \mathbf{E}(x,y,z) = \alpha \left[ (xy)^2 \hat{i} + (2yz) \hat{j} + (3xz) \hat{k} \right] \]

where \( \alpha \) is a constant with the appropriate units.

a) Which of these three vector functions can describe an electrostatic field?

b) For the vector function of part a) that can describe an electrostatic field find the corresponding electrostatic potential at point \( P(x, y, z) \), using the origin as your reference point.

c) Find the charge distribution that produces the electrostatic potential obtained in part b). Express all your answers in terms of \( \alpha, x, y, \) and \( z \).